MODELING THE CAPACITY OF LEFT-TURN AND THROUGH MOVEMENT CONSIDERING LEFT-TURN BLOCKAGE AND SPILLBACK AT SIGNALIZED INTERSECTION WITH SHORT LEFT-TURN BAY

A Thesis

by

KYOUNGMIN CHO

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2009

Major Subject: Civil Engineering

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Approved by:

Chair of Committee,	Yunlong Zhang
Committee Members,	Luca Quadrifoglio
	Thomas E. Wehrly
Head of Department,	David Rosowsky

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ABSTRACT

Modeling the Capacity of Left-Turn and Through Movement Considering Left-Turn Blockage and Spillback at Signalized Intersection with Short Left-Turn Bay.

(August 2009)

Kyoungmin Cho, B.S., Korea Military Academy, Seoul Korea

Chair of Advisory Committee: Dr. Yunlong Zhang

This research presents more realistic models for left-turn and through volume capacity by taking into account the probabilistic nature of the left-turn bay blockages and spillbacks at a signalized intersection under the leading phasing scheme with a short left-turn bay. Generally, the left-turn bay spillback situation has been overlooked in the leading left-turn signal because much attention has been given to the more common problem of left-turn blockage under the leading left signal. The left-turn spillback situation, however, might happen because the ratio of left-turning vehicle tends to be relatively high in the traffic after the occurrence of left-turn bay blockage. That is, left-turn bay blockage, spillback situations, left-turn capacity, and through capacity are closely connected with one another.

Hence, this research estimates more precisely the capacity for through and left-turn movement by considering the left-turn bay blockage and spillback situations associated with left-turn bay under leading left-turn signal operations. In order to find general agreement between the results from this proposed model and a real-world situation, the developed capacity model is validated with the results from CORSIM simulations of a real-world signalized intersection. The binomial distribution is applied as the arrival distribution for through movement considering the characteristics of expected arrivals under heavy flow conditions. Finally, since left-turn bay blockage and spillback situation seem to have adverse impacts on each other, this research investigates if there are any dependent relationships between left-turn bay blockage and spillback. Here, this study confirmed that close relationships between left-turn bay blockage and spillback situations obviously exists.

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CHAPTER I

INTRODUCTION

There have been many studies on left-turn treatments related to cycle length, phasing sequence and operational performance because those are some of the most essential elements for improving overall operation and safety at signalized intersections. The recent focus has been on when there is a high demand of through and left-turning volume during the peak hour. In high demand situations, overly long queues lead to spillbacks or blockage involving left-turning vehicles. The situations become even worse when the length of the left-turn bay is inadequate combined with certain signal timing situations.

The current Highway Capacity Manual (HCM) tends to overestimate the capacity of left-turn and through movement because the potential left-turn bay blockage or spillback due to short left-turn bay at a signalized intersection is neglected (HCM, 2000). Zhang and Tong developed a more accurate model for the left-turn and adjacent through capacity by considering the left-turn bay blockage or spillback which can occur under different left-turn signal operations such as leading and lagging sequences (Zhang and Tong, 2007). They modeled the left-turn capacity considering the probability of left-turn bay blockage under leading protected left-turn signal and the adjacent through capacity with the probability of left-turn bay spillback under lagging left-turn. In their analysis the arrival distribution is assumed to be Poisson (Zhang and Tong, 2007).

This thesis follows the style of *Journal of Transportation Engineering*.

It is, however, evident that the randomness in traffic patterns per each cycle tends to be reduced in the real-world as traffic volumes, left-turning and through vehicles, increase to saturation level (congested condition) (Rengaraju and Rao, 1995). That is, there is a high possibility that the variance is much less than the mean for expected arrivals under heavy flow conditions. Also, for the leading left-turn signal, there is a strong likelihood that left-turning vehicles spillback will take place during through(TH) green time under congested traffic condition even though leading phasing gives the right-of-way to left-turn (LT) movement. This situation occurs more frequently in the cycles immediately after the occurrence of initial LT blockage. It might be obvious that the occurring frequency of this situation increases as the length of LT bay is getting shorter. This is because the ratio of LT vehicle tends to be relatively high among the following traffic platoon after the occurrence of LT blockage. That is, it might be particularly true in that LT movement does not commonly discharge the average capacity (vehicles per cycle) due to the negative effect of blockage. When this situation occurs, the capacity of the adjacent through lane is negatively affected during the remaining green time with the spillback.

Hence, it is essential to develop more realistic model for left-turn and through volume capacity by taking into consideration a variety of situations under the leading phasing scheme. To do so, it is first needed to apply the binomial distribution instead of the Poisson distribution as the arrival distribution. Also, it is required to more accurately compute the probability that LT spillback occurs under leading phasing scheme in order to more realistically estimate the capacity of the adjacent through movement.

Additionally, it is very important to grasp if the relationship between the probability of blockage and spillback under leading phase sequence is.

Problem statement

The primary purpose of this study is to more accurately estimate the capacity of left-turn and adjacent through movement under left-turn leading signal when the status of traffic flow is congested. There have been recent studies that analyzed the change in the capacity when left-turn bay blockages and spillbacks situation occurred due to short left-turn bay. This study will determine the capacity more accurately by applying the binomial distribution which better reflects in congested traffic conditions in the real-world. If the probability of left-turn bay spillback into the adjacent through lane is obtained, the issue of traffic in the adjacent through lane may be better addressed. That is, it allows traffic engineers to estimate a more accurate capacity for the through movement. Furthermore, since left-turn bay blockage and spillback situation seem to have adverse impacts on each other, this research will be a more comprehensive study to ascertain if there are any dependent relationship between left-turn bay blockage and spillback.

Research objectives

The primary goal of this study is to more accurately model the capacity of left-turn and adjacent through movement under left-turn leading signal in traffic congestion which the left-turn bay blockage or spillback frequently take place. For this purpose, the following specific objectives should be achieved:

- To compare the binomial distribution with the Poisson one to assess which one is more suitable to describe the arrival pattern for each movement during peak hour;
- To model the left-turn capacity considering left-turn bay blockage under leading phasing scheme when the arrival distribution is a binomial distribution;
- To model the adjacent through capacity with left-turn bay spillback under leading phasing scheme when the arrival distribution is a binomial distribution;
- To demonstrate using CORSIM simulation program how well the capacity models reflect traffic operations with spillback and blockage related to insufficient LT bay length;
- To grasp the relationship between the probability of left-turn bay blockage and spillback at signalized intersection with a short left-turn bay;

Thesis organization

This thesis is composed of six chapters. The first section of the thesis addresses the background including the problem statement and research objectives. The second part of the thesis provides a review of previous research on left-turn operations, arrival distributions, and capacity estimation at a signalized intersection. The third section presents the data collection method and the data to select an appropriate arrival distribution. The fourth section describes the methodology such as statistical approach and modeling process for the capacity employed for this study. The section also illustrates the results from the data analysis and estimation of the capacity for LT and TH vehicles based on a developed model. Lastly, the sixth section states the executive summary of this research including findings, limitations and the needs for future work.

CHAPTER II

LITERATURE REVIEW

The author reviewed previous studies related to left-turn treatment as the preliminary background. The reviewed studies focused on left-turn operations, the appropriate length of left-turn bay, and applications of blockage and spillback concept for capacity enhancement. The initial review proved to be very beneficial and instrumental in the initiation of this thesis research.

Signalized intersection, left-turn operations and arrival distribution

Traffic signal is a very important form of intersection control. It can not only considerably increase the overall efficiency of traffic operation at the intersection but also reduce the safety problems such as crashes at the intersection by assigning the right-of-way to specific movements at a given time. As the traffic volume increases, there is no other form of control short of grade separation that can do as much as signals do (Roess, Prassas, and McShane, 2004).

Left-turns are the most difficult and complex procedures to deal with at a signalized intersection. There are several different ways of left- turns. Different design such as exclusive left-turn lane and shared lane (with through movement) is one of several elements that should be considered. A traffic signal can also have different left-turn phasing strategies: permitted left-turn, protected left-turn and permitted/protected left-turn depending on traffic situations such as volume, service capacity, and the intersection geometry design. Here, the appropriate treatments regarding left-turn is very essential to provide better operation and safety of the intersection according to several elements such as left-turning and opposing volume related to traffic operation(Roess et al., 2004). Roger et al. suggested general guidelines for selecting more suitable left-turn sequence in each intersection with various conditions. Permitted phase is acceptable when the number of crashes related to left-turn movement is fewer than eight and when there are no sight distance restrictions, and when the left-turn demand flow within the peak hour, against the speed limit for opposing traffic, falls within the permitted portion of the exhibit as shown in Figure 1.

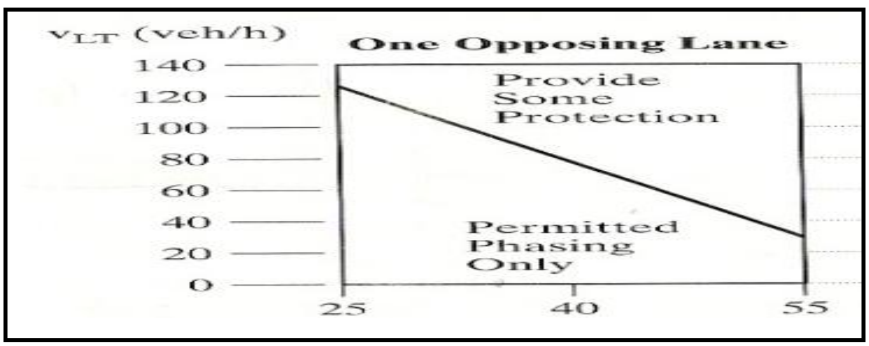

FIGURE 1 The standard for selecting the permitted phasing sequence (Roess et al., 2004)

Only protected phase is satisfied when it is satisfied at least two things of the following conditions (Roess et al., 2004):

- Left-turn flow rate is greater than 320 vph.
- Opposing flow rate is greater than 1,100 vph.
- Opposing speed is larger than 45 mph.

- Left-turn lanes are more than two lanes.

Also, the protected phase is satisfied when it is satisfied only one thing of the following conditions (Roess et al., 2004):

- Opposing traffic lane≥3 & Opposing speed≥45 mph
- Left-turn flow rate>320 vph
- Opposing flow rate>1,100vph
- Left-turn accidents≥7 (within 3 years under compound phasing)

Lastly, protected-permitted phase is selected when it satisfied at least one of the following things (Roess et al., 2004):

- v_{LT} (Left-turn flow rate) ≥ 200 vph
- $v_{LT}(v_0 / N_0)$ ≥ 50,000 (V_o is opposing flow rate.)
- Left-turn vehicle >2 vehicles per cycle

As shown above, there are some guidelines for selecting protected phase and protected-permitted phase (Roess et al., 2004). Especially, the modeling of permitted left-turn is required to analyze the complex interactions between permitted left-turn and the opposing flow of vehicles. That is, the number of lanes of opposing approach, arrival type, volume, and speed should be considered because these factors have a significant effect on the capacity of left-turn (Roess et al., 2004). It is also needed to obtain the actual green time for left-turn because the duration that is actually available to left-turning vehicles is a part of the total green time. In the HCM (HCM, 1994), the green time for the permitted left-turn is divided into three components defined as the following;

g_f: This is the portion of the green for a shared lane during which through vehicles move until the arrival of the first left-turning vehicle.

g_q: This is the portion of the green phase that is blocked by the clearance of an opposing queue.

g_u: This is the portion of the green phase not blocked by the clearance of an opposing queue. (HCM, 2000)

Since the vehicle arrivals are count data, various discrete distributions such as the Poisson and binomial distributions are applied to investigate the distribution of the data. V.R Rengaraju et al. found that the a Poisson distribution gives a close estimate to vehicle arrivals when the traffic volumes are less than 500 vehicles/hour/lane and for higher and mixed traffic volumes a multivariate distribution concept can be used (Rengaraju and Rao, 1995). It was first observed by Adams (1936) that the number of vehicles passing a point in equal intervals of time follows a Poisson distribution (Adams, 1936) and Schuhl (1955) applied probability theory to distribution of vehicles on two-lane highways (Schuhl, 1955).

In a Poisson distribution, the formula is $P(x) = \dfrac{e^{-m} \times m^x}{x!}$, and the value of x (the number of events = 0, 1, 2, $\cdots$, n) means the number of discrete events occurring during a time interval. Here, since this research is related to through movements and left-turning volume, the value of x is defined as how many cars intend to turn left or go through arrive at an intersection during a specified time interval. It is commonly suitable to describe the process as random arrival situation under a low level of traffic volume.

Similarly, the binomial distribution also shows how many cars could show up at the intersection. The equation for the binomial distribution is as follows:

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

where,

n = the number of total trials

x = the number of successes in n (x = 0, 1, 2, · · · , n)

p = the probability of a success in a single trial

It has a mean (which equals $n \times p$). So suppose the mean is 7, this means 7 left-turning or through cars are expected to arrive at the intersection during a unit time period.

On the other hand, the differences between two distributions obviously exist on traffic operations. Using the example mentioned above, although 7 cars are expected to arrive, how many will actually arrived in the real world? Probably, this answer is not exactly 7. Even if it was 7 left-turning or through cars as an average value, it will be sometimes 4 cars, sometimes 11 cars, sometimes no cars! So the difference in these distributions is how likely things that are difficult to expect will happen (Montgomery and Runger, 2006).

In a binomial distribution, the probability of having zero cars arriving at an intersection can also be calculated as P(0)=(1-p)^n. There is also a probability for having one car (the probability is $n \times p^1 \times (1-p)^{n-1}$) and there is a probability for two cars, three cars, and so on. Unlike the Poisson, however, the binomial distribution only has probabilities for up to n cars. After that there is no probability - basically saying it's not possible to have more than n cars show up at the intersection during a fixed time interval. This distribution could be good because it is realistic to assume that there is a limit to how many cars could arrive at this intersection due to the limitation of saturation flow rate. Also, it should be more appropriate to express the congested traffic situation because of its characteristic that the value of a mean is normally larger than the value of a variance (Montgomery and Runger, 2006).

Gattis (2000) depicted that engineers have to use statistical distributions to predict the turning volumes in heavy flows in order to determine the appropriate length of the turning lanes. He described that there are four distributions, namely Poisson, binomial, normal and uniform distribution, to describe vehicle arrivals. Gattis (2000) conducted experiments on three different turning lanes, with volumes ranging from 124 vph to 484 vph. Protected left-turn was allowed with three different cycle length: 100 s, 120 s or 140 s. He indicated that the binomial distribution is more appropriate than the Poisson distribution as an estimator of turning vehicles at 95th and 99th percentile levels; however, the binomial distribution is more difficult due to more requirements such as the determination of mean and variance. In a case in which a ratio of variance to mean was assumed to be 0.8 and the average arrival rate is larger than 5 to 7 vehicles per cycle, the binomial distribution approximately predicted the arrival rate that is less than the arrival rate of Poisson distribution. Gattis determined the required left-turn storage length according to average number of vehicles per cycle and vehicle storage needed to be adequate 95% of the time (Gattis, 2000).

A goodness-of-fit test is a hypothesis test regarding the distribution of a population. It determines differences between the sample distribution and a theoretical distribution (Massey, 1951). There are some tests for goodness-of-fit including the Chi-square test and K-S test. The Chi-square test is the test following a Chi-square distribution as one of the statistical hypothesis tests. Another test is the K-S test which judges goodness-of-fit by using population cumulative distribution. The maximum absolute difference (D) is then calculated between observed and theoretical value, and later compared with the critical value.

Massey (1951) developed an alternative distribution-free test of goodness-of-fit and presented evidence indicating that when it is applicable, it may be a better all-around test than the Chi-square test. The comparison between the chi-square and K-S test is below as shown in Table 1.

TABLE 1 Goodness-of-fit tests and respective strengths(S) and weaknesses(W)

	Chi-square test	K-S test
Power of test	W: unknown	S: A lower bound to the power of test is known
Loss of sample's information	W: loss by grouping	S: No loss because of treating individual observation separately
Computing time	-	S: Less than chi-square test
visualization	W: No graph	S: Graphical test can be used
Degree of freedom modification	W: Applicable	S: Not applicable
Apply to discrete population	S: Applicable	W: Not applicable

First, while the power of the chi-square is not known, the lower bound to the power of the K-S test is known. Furthermore, the K-S test treats individual observations separately, thus does not lose information by grouping. The K-S test usually requires less computation than chi-square test. Some disadvantages of using the K-S test versus the chi-square test are also evident. The chi-square test is easily modified by reducing the number of degrees of freedom; the K-S test can not easily be modified. The K-S test also cannot be applied to a discrete population, while the chi-square test can be (Massey, 1951).

Another way to test goodness-of-fit test is a Q-Q plot which is a graphical method for diagnosing differences between the model distribution and the observed distribution. A Q-Q plot provides a good way to visualize the goodness-of-fit for each model. In addition, there are several tests for goodness-of-fit, similar to the K-S test including the Lilliefors test, the Jarque-Bera test and the Kuiper's test. The Lilliefors test and the Kuiper's test are derived from the K-S test (Massey, 1951).

Determination of the appropriate storage length

Harmelink (1967) studied about the volume warrants for left-turn storage at unsignalized at-grade intersections on four-lane (divided and undivided) and two-lane highways by dividing three categories such as theoretical analysis, a series of field studies of traffic behavior, and analysis of a series of questionnaires. For theoretical analysis, queuing theory was used, and the arrival and service rate of left-turning vehicles were assumed to follow a Poisson distribution. In this analysis, the need for additional left-turn storage lane was generally determined if the safety and capacity of the through lane were affected by the presence of left-turning vehicles. The arrival rate on four-lane highways was defined as the number of vehicles per hour making left-turns, and the arrival rate on two-lane highways was determined by using several factors such as the volumes of left-turning, through and opposing traffic, and the time interval needed for left-turning. Service rate of both highways was made a decision by the number of left-turns that are able to be conducted in one hour by defining unblocked time. In order to determine the values of various parameters applied in this analysis, field studies were performed at seven Ontario intersections. Through analysis of questionnaires, it was known that an engineer's judgment as well as volume conditions might be needed for analysis of intersections with poor visibility and / or a bad accident record (Harmelink, 1967).

Kikuchi, Kii, and Chakroborty (2004) stated that length of dual left-turn is an important design factor. He developed a procedure to determine the lane length for preventing both the lane overflow and lane blockage. First, the procedure develops a relationship between lane use and volume of left-turn vehicles. Second, the procedure formulates the probability of all the left-turning vehicles entering the left-turn lane during the red phase. Third, the adequate length of the lane is expressed in terms of the number of vehicles. Left-turn bay lengths are recommended based on left-turn, and through volumes in order to avoid lane overflow and blockage of lane entrance (Kikuchi et al., 2004).

Kikuchi, Kronprasert, and Kii (2007) also studied the appropriate length of turn lanes when single lane is split into three lanes such as left-turn, through, and right-turn at a signalized intersection (Kikuchi et al., 2007). To do so, he conducted the following procedures. Firstly, he defined several parameters (total approach volume, proportion of turn volume, the duration of red phase, etc) related to the determination of the length for turn lane and assumed their viable values. Secondly, he built up the patterns (8 cases related to blockage or overflow) of vehicle arrival at the end of the red phase and the expression of probability (Poisson distribution was assumed) for each arrival pattern. Lastly, he obtained the length of the lane that meets the borderline probability of acceptable condition. As a result, he found out that this method regarding the three lanes just needs a shorter turn lane than the original one based on the AASHTO Green Book and the guidelines of state DOTs. This is because it reduces the chances of lane overflow and blockage by splitting the arriving vehicles into three directions. He said it is particularly true when the volume of each movement is almost even (Kikuchi et al., 2007).

Qi, Yu, Azimi, and Guo (2007) developed a new method to more precisely estimate the storage lengths of left-turn lanes at signalized intersections. The remarkable difference between their new model and the other existing methods is to consider the leftover queue at the end of green time as well as vehicles arriving during red time. They established model for estimation of queue formed during red time by using the Poisson distribution and applied the Discrete-Time Marcov Chain to determine the leftover queue at the end of green time (Qi et al., 2007). The specific equations are as follows:

The first one is related to queue formed during red phase (Qi et al., 2007):

$$\text{Prob(arrivals in red phase} < k) = \Pr ob(k) = \frac{(\lambda_t R)^k \times e^{-\lambda_t R}}{k!}$$

where,

λ_t = average arrival rate of left-turning vehicles [vehicles per second]

R = duration of red phase

Here, the maximum number (Q_1) of vehicles arriving during the red phase can be determined by the following equation when the required probability level (α_1) is given.

$$\text{Prob(arrivals in red phase} < Q_1) = \sum_0^{Q_1} \Pr ob(k) = \sum_0^{Q_1} \frac{(\lambda_t R)^k \times e^{-\lambda_t R}}{k!} = \alpha_1$$

Secondly, the transition matrix (P) should be obtained to estimate the leftover queue. In order to get all individual elements P_{ij} of the transition matrix, the following three situations need to be considered (Qi et al., 2007):

1st situation: There is no leftover queue at the next cycle by discharging all the vehicles in the queue in the current cycle.

$$\mathrm{p_{ij}} = \text{Prob(arrivals in cycle} \leq m - i) = \sum_0^{m=i_1} P_k^C$$

2nd situation: Left-turning queue carryover will take place in the next cycle.

$$\mathrm{p_{ij}} = \text{Prob(arrivals in cycle} = m + j - i) = P_{m+j-i}^C$$

3rd situation: m vehicles in one cycle are the maximum number that can be discharged in one cycle.

$$\mathrm{p_{ij}} = 0$$

where,

i = the number of leftover queue vehicles at the current time

j = the number of leftover queue vehicles at the current time

m = intersection service rate

$$\mathrm{P}_{\mathrm{k}}^{\mathrm{C}} \mathrm{Prob}(\text{arrivals in } cycle = k) = \frac{(\lambda_t R)^k \times e^{-\lambda_t C}}{k!}$$

They defined the upper bound of the leftover queue (φ), and for getting the stationary probability of i vehicles carry over at the end of green phase (πi), P matrix was multiplied by the DTMC stationary-probability row vector (π). Hence, the maximum leftover queue length, Q_2 can be obtained by the following equation when the required probability level (α_2) is given. $\mathrm{Prob}(\text{number leftover of vehicles} < Q_2) = \sum_{i=1}^{Q_2} \pi_i = \alpha_2$

Based on these procedures and concepts, they showed that this new method is a better estimator for the appropriate length of left-turn lane than any other existing methods by evaluating this new model and comparing with other models (Qi et al., 2007).

The probability of blockage and spillback, estimation of the capacity

Levinson and Prassas (2001) compared the left-turn capacities of shared lane from four methods such as HCM and Canadian, SIDRA and Levinson methods for varying cycle length, through and left-turning volumes, etc. They found that increasing the cycle length from 60s to 90s decrease approach capacities slightly when the effective green ratio is assumed to be same. However, since there is less time lost per hour in the case of longer cycle length, it generally achieves a slight increase in capacity for same total green per cycle ratio (Levinson and Prassas, 2001).

Figure 2 shows that the percentage time of left-turn bay overflow consistently decreases as the lane length for left-turn increases. Herein, Lakkundi, Garber and Fontaine (2004) stated that a desired lane length for left-turn could be determined for the candidate intersection by defining an acceptable probability of left-turn lane overflow by using this feature in LTGAP. In other words, left-turn lane guidelines cannot be economically justified with only vehicle delay savings or the increase of capacity because the cost of construction is very high, and delay savings or the increase of capacity can be so small (Lakkundi et al., 2004).

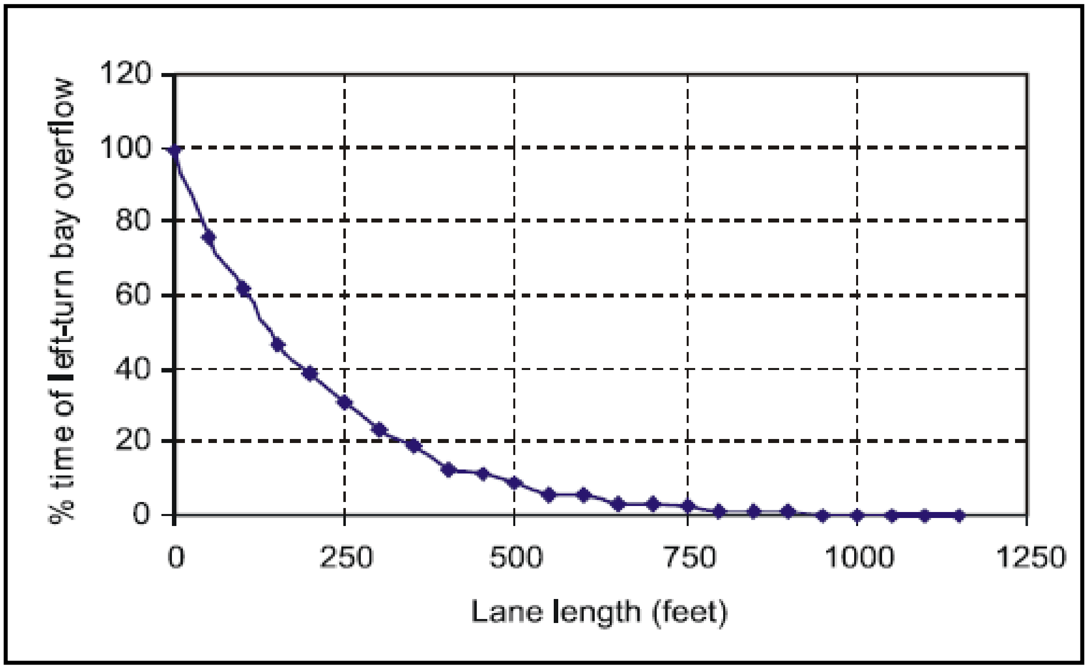

FIGURE 2 The percentage of left-turn bay overflow with lane length (ft)

Zhang and Tong (2007) suggested a new left-turn and adjacent through capacity model by including the capacity that is reduced due to left-turn bay blockage or spillback during peak hour (Zhang and Tong, 2007). In their study, the blockage situation takes place when there is no left-turn bay spillback before (N+2)th through vehicle arrives on the adjacent through lane. Also, adjacent through lane blockage by left-turn bay spillback occurs due to the same reason. In order to explain this situation and calculate the probability for a blockage situation like this one, they used a Negative Binomial distribution and made an assumption that the count distribution of through movement follows a Poisson distribution (Zhang and Tong, 2007).

Additionally, they assessed the effects on the probability of blockage according to the length of left-turn bay and left-turn signal strategy. They found out that the length of the left-turn bay is very sensitive to the left-turn and adjacent through capacity during peak hour because the probability of left-turn bay blockage or spillback is significantly affected. And they recommended that the lagging protected left-turn phasing is more appropriate when a massive adjacent through volumes and a short left-turn bay exist because it is efficient to first discharge through vehicles to avoid the blockage of the entrance to the left-turn bay. Also, they developed and designed the adjacent through and the left-turn capacity model which better reflects the real-world than HCM method by considering the capacity loss due to left-turn bay blockage or spillback (Zhang and Tong, 2007).

Previously, Zong Z. Tian et al. (2006) studied a capacity estimation model for a signalized intersection with a short right-turn lane in an almost same way (Tian and Wu, 2006). They also established the appropriate model for getting the probability of blockage and spillback, and then estimated the capacity for through and right-turn movement considering these factors. They found that the capacity of a signalized intersection with a short right-turn lane is significantly affected by several elements such as the length of right-turn lane, cycle length, and the proportion of through and right-turn vehicles (Tian et al., 2006).

CHAPTER III

DATA COLLECTION AND DESCRIPTION

Vehicle arrival data were collected to investigate the arrival distributions for through vehicles and left-turning vehicles. These data were produced from both a real-world intersection and also from CORSIM simulation program.

Data collection method for selection of the arrival distribution using simulation

First, the appropriate data that can realistically cause left-turn bay blockage and spillback situations were fed into the CORSIM program. These data basically includes geometry design, approaching volume, post speed limit and signal timing data, and were utilized to compute the probability of left-turn bay blockage and spillback. The key input data, which is based on a real-world intersection, for this simulation case are summarized in Table 2.

TABLE 2 Key input data for this research

Key Input Data for the Approach Studied	TH Volume (veh/h)	LT Volume (veh/h)	TH Green (sec)	TH Red (sec)	Protected LT Green (sec)	Opposing TH Arrival Type	Opposing TH Volume (veh/h)	TH Saturation Flow Rate (veh/h/ln)	Protected LT Saturation Flow Rate (veh/h/ln)
Values	1600	400	52	68	19	3	1000	1850	1800

This study will provide a reliable proof for selecting proper distributions for vehicle movements based on data. Actual values of mean and variance will also be obtained through the data provided from CORSIM program. Basic units for these parameters will be used as vehicles per 10 seconds. That is, it is necessary to find out how many vehicles, on average, arrive at the designated intersection in ten seconds. Here, left-turning and through vehicles should be investigated seperately. To make the data more reliable, the sample size of 360 intervals (during 1 hour) will be collected by using the surveillance detector function in CORSIM. To do so, two presence detectors were installed in the upstream of the designated intersection. The specific location of these detectors is as the following Figure 3:

FIGURE 3 The location of detector and the overall outline of the designated intersection

Here, the length of left-turn bay is 400ft, and the two detectors were located 350ft upstream from the stop line of the intersection. That is, in order to objectively analyze the arrival pattern of through and left-turning cars, the two detectors and the stop line must be apart far enough so that vehicles arriving at the location of the detectors would not be interrupted by the spillback situation or original queues.

Also, since the detector is only able to count the total number of vehicles including left-turning and through volume passing by it, two detectors are needed at the same spot. So one detector covers all lanes including two through lanes and left-turn bay, to figure out the total number of passing vehicles. The other one covers every full lane except the left-turn bay in order to enumerate the number of passing through vehicles. In other words, the left-turn volume can be calculated through the difference between the outputs of the two detectors.

By using these detectors, the data for selecting the arrival distribution were collected. It was investigated how many through and left-turning vehicles arrive at the designated intersection per 10 seconds, respectively. Generally, the larger the sample size (n), the greater the accuracy that the results from these samples truly reflect the population. This is because sample size is mainly affected by several factors such as confidence level and interval. In other words, varying the sample size is the easiest method to affect the confidence interval width for maintaining the required confidence level. So, it is reasonable that the sample size of 360 was collected.

Here, arrival data were collected by dividing two kinds of traffic condition such as a low range and a high range in order to grasp the difference of an arrival distribution according to the traffic condition. The input is as follows:

1) Input for traffic condition of a high range is the same as Key input data for this research (Table 2).

2) Input for traffic condition of a low range:

Key Input Data for the Approach Studied	TH Volume (veh/h)	LT Volume (veh/h)	TH Green (sec)	TH Red (sec)	Protected LT Green (sec)	Opposing TH Arrival Type	Opposing TH Volume (veh/h)	TH Saturation Flow Rate (veh/h/ln)	Protected LT Saturation Flow Rate (veh/h/ln)
Values	800	200	50	44	17	3	1000	1850	1800

Specific arrival data for both conditions were summarized in APPENDIX A.

Through these arrival data, the value of mean and variance can be calculated using the following equations because those values are needed to select a better distribution for describing this observed data (Montgomery and Runger, 2006). The results are shown in the following Table 3:

$$\text{Mean:} \quad \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{1}$$

$$\text{Variance:} \quad s^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1} \tag{2}$$

where,

x_i = i_{th} observation from Random variable of size N

N = Sample size

TABLE 3 The value of mean and variance for low and high traffic conditions

	High range of volume (2000vph)		Low range of volume (1000vph)	
	Through	Left-turn	Through	Left-turn
Mean	4.46	0.87	2.24	0.31
Variance	2.89	0.91	2.04	0.29

Real-world intersection data collection for selection of the arrival distributions

This study is very essential in order to prove if the data provided from CORSIM are indeed trustworthy for determining of the arrival distribution and for using in this research. Real-world data were collected at the intersection of Texas Avenue and Holleman Drive in the city of College Station, Texas, and basic units for these arrival data were used as vehicles per 10 seconds. Here, the sample size of 238 intervals was collected by using one digital camera, and the digital camera was installed in the upstream (on southbound Texas Avenue) of the designated intersection. Also, real-world arrival data were collected during the afternoon peak hour (17:30~18:30) on 30th of April in 2009. The specific location of this digital camera is as the following Figure 4:

FIGURE 4 The location of digital camera and the overall outline of the real-world intersection

Texas Avenue consists of three lanes for through movement and an exclusive left-turn lane that the length is 300ft. Here, the number of passing through vehicles on the outside through lane was excluded from the real-world data collection because these through traffics were mainly interrupted by vechicles that enter or leave HEB grocery store. And real-world data were collected without the distinction between left-turn and through vehicles, and the specific real-world arrival data were summarized in APPENDIX B.

Through these real-world arrival data, the value of mean and variance can be calculated using the Eq. (1) and Eq. (2) (Montgomery and Runger, 2006). The results are indicated in the following Table 4:

TABLE 4 The value of mean and variance for real-world arrival data

	Real world arrival data
Mean	4.75
Variance	2.98

CHAPTER IV

PROBABILITIES OF LEFT-TURN BLOCKAGE AND SPILLBACK WITH SHORT LEFT-TURN BAY

This chapter provides reliable probabilities of left-turn bay blockage and spillback at a signalized intersection under protected leading left-turn signal. To do so, Chi-Square test is first conducted to determine the most suitable distribution in heavy and light traffic conditions. And then, the probability that each number of left-turning and through vehicles arrive at the intersection during a designated time is computed using the most adequate distribution for left-turn and through vehicles under congested traffic condition. Also, this study presents all variables, equations, and conditions defined as the blockage and spillback required to more precisely calculate the probability of the left-turn bay blockage and spillback situations. Finally, this study demonstrates result of the probabilities according to the length of left-turn bay.

Estimation of arrival distribution for through and left-turn vehicles

It is very essential to select and apply the most appropriate distribution in various traffic conditions including congested and normal condition. That is, based on the objective of this research, the most suitable distribution under congested traffic condition should be determined and applied to more precisely produce the probability of left-turn bay blockage and spillback situation. Generally, the binomial distribution is known as a good estimator for a traffic condition with a high volume because variance compared to mean decreases as the traffic volume increases. It is, however, true that there have not been studies using the binomial distribution for analysis regarding several MOEs (Measure Of Effectiveness) in a high level of traffic condition. So there is a need to validate if the binomial distribution is indeed more fit than any other ones to describe the arrival distribution of congested traffic condition.

Hence, a chi-square test was respectively conducted regarding the Poisson and the binomial distribution to determine more appropriate distribution according to arrival patterns of each movement. It is very easy to apply the Poisson distribution because only one parameter (λ), the mean, required for this distribution is already obtained. However, in order to apply the binomial distribution, the value of n and p should be estimated from the sample mean and the sample variance. The equations for this esimation are as follows based on the method of moments:

$$\hat{p} = \frac{(m - s^2)}{m} \tag{3}$$

$$\hat{n} = \frac{m}{p} = \frac{m^2}{m - s^2} \tag{4}$$

where,

n = the total number of different sequences that contains x successes and n-x failures

p = the probability of success on a single trial

In other words, the probability that the X vehicles of left-turning or through movement arrive at the intersection according to the binomial distribution can be computed by using the outputs of these equations. Here, there is a high possibility that a natural number doesn't come out as the value of n. In this case, it is assumed that the number of n rounds off to the nearest whole number. And the value of p should also be changed for a fixed value of mean according to the variation of n due to this assumption.

The value of n and p obtained from this process are summarized in Table 5:

TABLE 5 The summary for the value of n and p related to the binomial distribution

	High range of volume (2000vph)		Low range of volume		Real-world Data
	Through	Left-turn	Through	Left-turn	
n	12.7 => **13**	Negative number	24.8 => **25**	4.7 => **5**	12.7 => **13**
p	0.35 => **0.34**	Negative number	**0.09**	0.07 => **0.06**	**0.37**

The confidence level of 95% (the level of significance (α) = 0.05) was assumed and the time interval to count arriving through vehicles was 10 seconds. The hypothesis, the Chi-Square value, and the rejection region for this test are as follows:

Null hypothesis (H_0) : There is no difference between the observed distribution and the Poisson (or Binomial) one.

Alternative hypothesis (H_A) : There is a difference between the observed distribution and the Poisson (or Binomial) one.

Chi-Square value:

$$\chi^2{}_{calc} = \sum_{i=1}^{N} \frac{(f_o - f_t)^2}{f_t} \tag{5}$$

where,

N = The number of net categories

f_o = Observed frequencies

f_t = Theoretical frequencies

Rejection region (for level α test) :

$$\chi^2{}_{calc} > \chi^2{}_{\alpha,\text{degree of freedom}}$$

where,

Degree of freedom = N-1-p

(p= the number of estimated parameters for each distribution)

Also, the Chi-Square test was performed by dividing two conditions including a high range of volume (total: 2000vph) and a low range of volume (total: 1000vph) based on the data obtained from CORSIM. This process provided clues about the suitable distribution for through and left-turn movement according to the different volume conditions. Especially, a histogram for the through movement in heavy traffic conditions was suggested to more clearly display which theoretical distribution fits well the observed distribution.

In addition, the Chi-Square test was conducted using the real-world data. This result supports that the data produced from CORSIM are reliable and confident, and that the binomial distribution is a better estimator of arrival distribution than the Poisson distribution in the real-world. Here, a histogram regarding real-world data is also displayed.

The test results for through movement with a volume of 1600vph (heavy traffic condition) are shown in the following Tables 6 and 7.

TABLE 6 The test results for through movement counts (1600vph)(Observed Vs. the Poisson)

# vehicles in interval	Observed Frequency	Poisson Probability	Theoretical Freq. for Poisson	Chi-square for Poisson
0	12	0.01	22.65	5.01
1		0.05		
2	28	0.11	41.31	4.29
3	60	0.17	61.47	0.04
4	89	0.19	68.60	6.07
5	76	0.17	61.24	3.56
6	59	0.13	45.56	3.96
7	20	0.08	29.06	2.82
8	11	0.05	16.21	1.68
9	5	0.02	13.90	5.70
10		0.01		
11+		0.01		
Total	360	1	360.00	33.12

N = 9 because (0, 1) and (9, 10, 11+) were grouped as one to maintain a minimum frequency of 5, respectively. So the degree of freedom is 7 because the number of parameter in the Poisson distribution is one, and $\chi^2_{0.05,7}$ is 14.1. Since $\chi^2_{calc} = 33.12$ (> 14.1), H_0 can be rejected. Therefore, it can be concluded that observed distribution is different from the Poisson distribution when the level of significance (α) is 0.05.

TABLE 7 The test results for through movement counts (1600vph)(Observed Vs. the Binomial)

# vehicle in interval	Observed Frequency	Binomial Probability	Theoretical Freq. for Binomial	Chi-square for Binomial
0	12	0.00	11.84	0.00
1		0.03		
2	28	0.09	32.39	0.60
3	60	0.17	62.11	0.07
4	89	0.23	81.20	0.75
5	76	0.21	76.43	0.00
6	59	0.15	53.29	0.61
7	20	0.08	27.87	2.22
8	16	0.03	14.87	0.09
9		0.01		
10		0.00		
11+		0.00		
Total	360	1.00	363.94	4.34

For the hypothesis test of Binomial distribution summarized in Table 7, N = 8 because (8, 9, 10, 11+) and (0, 1) were grouped as one to maintain a minimum frequency of 5, respectively. So the degree of freedom is equal to 5 because the number of parameters for the binomial distribution is 2. Hence, $\chi^2_{0.05,5} = 11.1$.

Since $\chi^2_{calc} = 4.34 < 11.1$, H_0 can not be rejected. Therefore, it can be concluded that observed distribution is not different from the binomial distribution when the level of significance (α) is 0.05.

Also, the histogram for through movement counts in heavy traffic condition is shown in Figure 5, and this histogram is obviously indicated to support the result of the chi-square test.

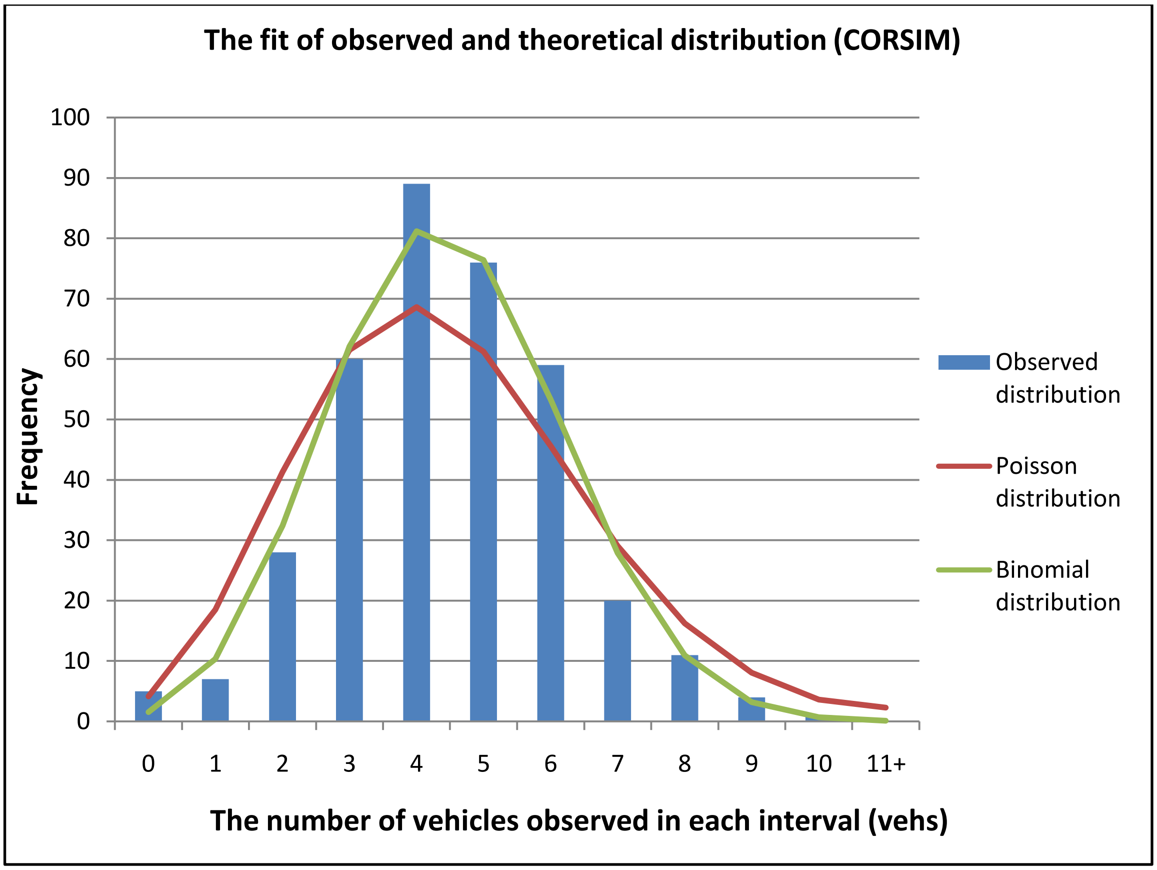

FIGURE 5 The histogram for through movement vehicles in heavy traffic condition (CORSIM)

The test results for through movement with a volume of 800vph (light traffic condition) are summarized in the following Tables 8 and 9.

TABLE 8 The test results for through movement counts (800vph)(Observed Vs. the Poisson)

# vehicle in interval	Observed Frequency	Poisson Probability	Theoretical Freq. for Poisson	Chi-square for Poisson
0	37	0.11	38.26	0.04
1	86	0.24	85.77	0.00
2	87	0.27	96.13	0.87
3	82	0.20	71.83	1.44
4	48	0.11	40.26	1.49
5	13	0.05	18.05	1.41
6	7	0.02	9.70	0.75
7		0.01		
8+		0.00		
Total	360	1.00	360.00	6.00

N = 7 because (6, 7, 8+) were grouped as one to maintain a minimum frequency of 5. So the degree of freedom = 7-1-1 = 5, and $\chi^2_{0.05,5} = 11.1$. Since $\chi^2_{calc} = 6.00$ < 11.1, H_0 can not be rejected. Therefore, it can be said that observed distribution is not different from the Poisson distribution when the level of significance (α) is 0.05.

TABLE 9 The test results for through movement counts (800vph) (Observed Vs. the Binomial)

# vehicle in interval	Observed Frequency	Binomial Probability	Theoretical Freq. for Binomial	Chi-square for Binomial
0	37	0.10	34.38	0.20
1	86	0.24	84.66	0.02
2	87	0.28	100.07	1.71
3	82	0.21	75.57	0.55
4	48	0.11	40.94	1.22
5	13	0.05	16.94	0.91
6	7	0.02	7.45	0.03
7		0.00		
8+		0.00		
Total	360	1.00	360.00	4.63

N = 7 because (6, 7, 8+) were grouped as one to maintain a minimum frequency of 5. So the degree of freedom = 7-2-1 = 4, and $\chi^2{}_{0.05,4} = 9.5$. Since $\chi^2{}_{calc} = 4.63 < 9.5$, H_0 can not be rejected. Therefore, it can be concluded that observed distribution is not different from the binomial distribution when the level of significance (α) is 0.05.

Next, the test result for left-turn movement with a volume of 400vph (heavy traffic condition) is indicated in the following Table 10.

TABLE 10 The test results for left-turn movement counts (400vph) (Observed Vs. the Poisson)

<table>
<tr><th># vehicle in interval</th><th>Observed Frequency</th><th>Poisson Probability</th><th>Theoretical Freq. for Poisson</th><th>Chi-square for Poisson</th></tr>
<tr><td>0</td><td>149</td><td>0.42</td><td>150.91</td><td>0.02</td></tr>
<tr><td>1</td><td>139</td><td>0.36</td><td>131.20</td><td>0.46</td></tr>
<tr><td>2</td><td>51</td><td>0.16</td><td>57.04</td><td>0.64</td></tr>
<tr><td>3</td><td rowspan="4">21</td><td>0.05</td><td rowspan="4">20.85</td><td rowspan="4">0.00</td></tr>
<tr><td>4</td><td>0.01</td></tr>
<tr><td>5</td><td>0.00</td></tr>
<tr><td>6</td><td>0.00</td></tr>
<tr><td>Total</td><td>360</td><td>1.00</td><td>360.00</td><td>1.13</td></tr>
</table>

N = 4 because (3, 4, 5, 6) were grouped as one to maintain a minimum frequency of 5. So the degree of freedom = 4-1-1 = 2, and $\chi^2_{0.05,2} = 6.0$. Since $\chi^2_{calc} = 1.13 < 6.0$, H_0 can not be rejected. Therefore, it can be said that observed distribution is not different from the Poisson distribution when the level of significance (α) is 0.05.

Here, it is not possible for the relationship between the observed distribution and the binomial distribution to be found because the value of mean (0.87) is smaller than the variance (0.91). This is because the value of p (the probability of success) comes out negatively based on the estimated equation of p.

The test results for left-turn movement with a volume of 200vph (light traffic condition) are shown in the following Tables 11 and 12:

TABLE 11 The test results for left-turn movement counts (200vph) (Observed Vs. the Poisson)

# vehicle in interval	Observed Frequency	Poisson Probability	Theoretical Freq. for Poisson	Chi-square for Poisson
0	261	0.73	263.02	0.015
1	85	0.23	82.56	0.072
2+	14	0.04	14.43	0.013
Total	360	1.00	360	0.10

Since N = 3, the degree of freedom = 3-1-1 = 1. Hence, $\chi^2_{0.05,1} = 3.84$. Since $\chi^2_{calc} = 0.1 < 3.84$, H_0 can not be rejected. Therefore, there is no sufficient evidence that observed distribution is different from the Poisson distribution when the level of significance (α) is 0.05.

TABLE 12 The test results for left-turn movement counts (200vph) (Observed Vs. the Binomial)

# vehicle in interval	Observed Frequency	Binomial Probability	Theoretical Freq. for Binomial	Chi-square for Binomial
0	261	0.72	260.32	0.00
1	85	0.24	87.19	0.05
2+	14	0.03	11.68	0.46
Total	360	1.00	359.19	0.52

It is not possible for this case, shown in Table 11, to be analyzed using the Chi-Square test because the degree of freedom is zero (3-2-1=0) even though the value of mean is greater than the variance.

The test results for real-world arrival data during afternoon-peak hour (heavy traffic condition) are indicated in the following tables 13 and 14:

TABLE 13 The test results for real-world arrival data (Observed Vs. the Poisson)

<table>
<tr><th># vehicles in interval</th><th>Observed Frequency</th><th>Poisson Probability</th><th>Theoretical Freq. for Poisson</th><th>Chi-square for Poisson</th></tr>
<tr><td>0</td><td rowspan="2">6</td><td>0.01</td><td rowspan="2">11.86</td><td rowspan="2">2.90</td></tr>
<tr><td>1</td><td>0.04</td></tr>
<tr><td>2</td><td>16</td><td>0.10</td><td>23.26</td><td>2.26</td></tr>
<tr><td>3</td><td>34</td><td>0.15</td><td>36.81</td><td>0.21</td></tr>
<tr><td>4</td><td>49</td><td>0.18</td><td>43.69</td><td>0.65</td></tr>
<tr><td>5</td><td>56</td><td>0.17</td><td>41.49</td><td>5.08</td></tr>
<tr><td>6</td><td>42</td><td>0.14</td><td>32.83</td><td>2.56</td></tr>
<tr><td>7</td><td>18</td><td>0.09</td><td>22.27</td><td>0.82</td></tr>
<tr><td>8</td><td rowspan="2">17</td><td>0.06</td><td rowspan="2">25.80</td><td rowspan="2">3.00</td></tr>
<tr><td>9+</td><td>0.05</td></tr>
<tr><td>Total</td><td>238</td><td>1</td><td>238</td><td>17.48</td></tr>
</table>

N = 8 because (0, 1) and (8, 9+) were grouped as one to maintain a minimum frequency of 5, respectively. So the degree of freedom is 6 because the number of parameter in the Poisson distribution is one, and $\chi^2_{0.05,6}$ is 12.6. Since $\chi^2_{calc} = 17.48$ (> 12.6), H_0 can be rejected. Therefore, it can be concluded that observed distribution is different from the Poisson distribution when the level of significance (α) is 0.05.

TABLE 14 The test results for real-world arrival data (Observed Vs. the Binomial)

# vehicles in interval	Observed Frequency	Poisson Probability	Theoretical Freq. for Poisson	Chi-square for Poisson
0	6	0.003	5.48	0.05
1		0.020		
2	16	0.070	16.70	0.03
3	34	0.148	35.22	0.04
4	49	0.213	50.66	0.05
5	56	0.220	52.47	0.24
6	42	0.169	40.25	0.08
7	18	0.097	23.16	1.15
8	17	0.042	14.05	0.62
9+		0.017		
Total	238	1	238	2.26

Here, N = 8 because (8, 9+) and (0, 1) were grouped as one to maintain a minimum frequency of 5, respectively. So the degree of freedom is equal to 5 because the number of parameters for the binomial distribution is 2. Hence, $\chi^2_{0.05,5} = 11.1$

Since $\chi^2_{calc} = 4.34 < 11.1$, H_0 can not be rejected. Therefore, it can be said that observed distribution is not different from the binomial distribution when the level of significance (α) is 0.05.

Also, the histogram for real-world data during the peak hour is displayed in Figure 6, and this histogram also supports the result of the chi-square test regarding real-world data.

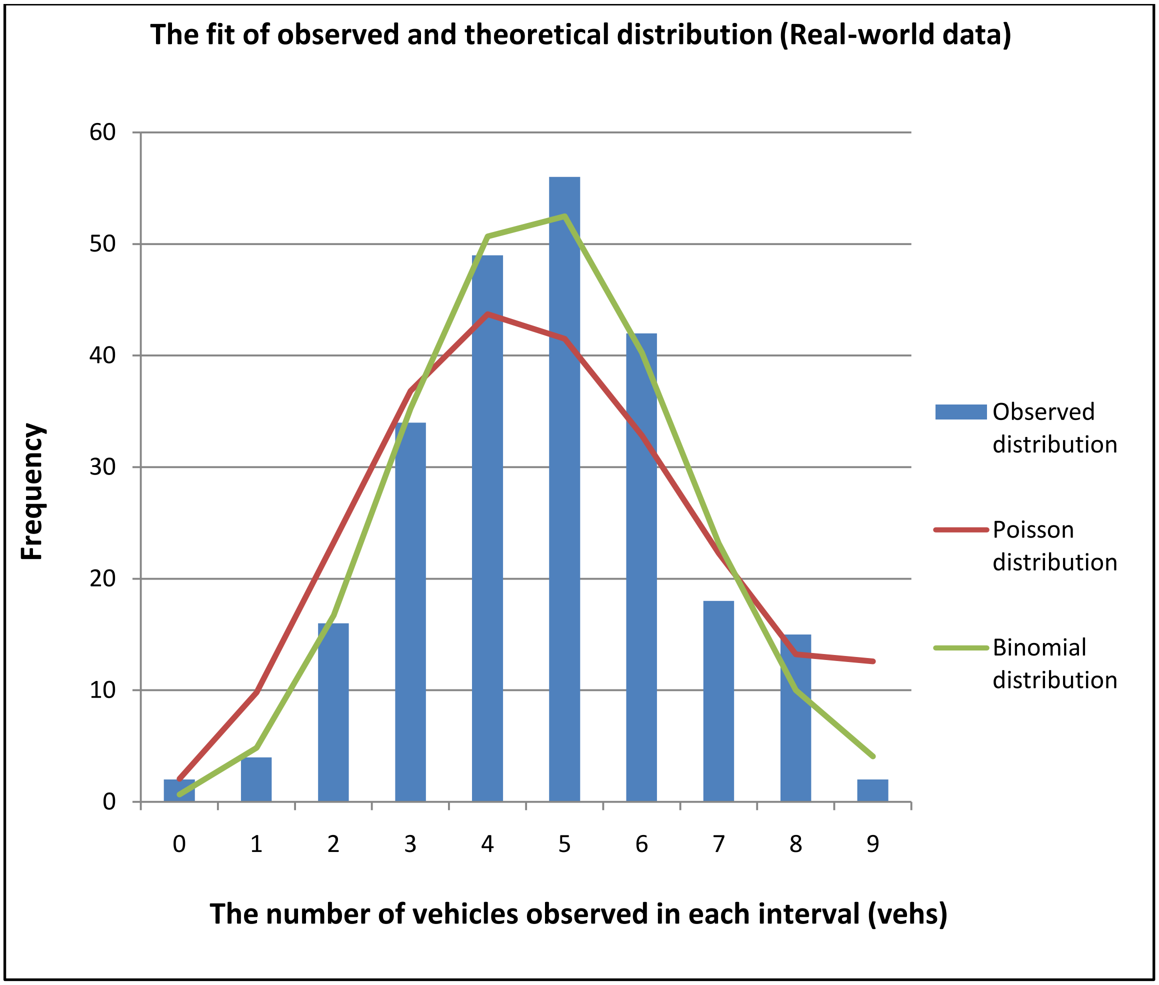

FIGURE 6 The histogram for real-world data during the peak hour

Table 15 presents the Summary of the Chi-Square test results:

TABLE 15 Summary of the Chi-Square test results

	High range of volume		Low range of volume		Real-world arrival data
	Through	Left-turn	Through	Left-turn	
Poisson	Different	Not different	Not different	Not different	Different
Binomial	Not different	Impossible	Not different	Impossible	Not different

As it was expected, the result shows that the Poisson distribution is more appropriate for the arrival distribution of left-turn movement regardless of traffic condition. This is because left-turning vehicles tend to almost randomly arrive, in that left-turn volume is commonly small, thereby increasing the variance rather than the mean. Also, the result indicates that both arrival distributions for the arrival data obtained from CORSIM and the real-world arrival data with a high volume are more reasonably described as the binomial distribution based on Chi-Square value. Namely, this is particularly true, in that the variance is significantly smaller than the mean because the randomness of arrival is reduced with the increase of volume.

However, it is required to examine more carefully the result for through movement in a low volume of traffic condition. This result could be controversial because both distributions are not different from the observed distribution within 95% of confidence level. Even though it is difficult to judge the more appropriate distribution using the result of the Chi-Square test, the Poisson distribution would be more desirable for through movement in a light traffic condition because the mean (2.24) and variance (2.04) are almost the same.

Calculation for the probability of LT bay blockage and spillback

The leading left-turn phasing sequence is a way to give a priority to the left-turning vehicles. Left-turn bay spillback situation can take place due to the left-turn queues accumulated in the middle of the green time for through movement in addition to original left-turn queues. Also, left-turn bay blockage situation due to through vehicles can occur because queuing through vehicles exceeding the length of the left-turn bay are sometimes formed during the red time for through movement. Since the capacity for leading left-turn signalized intersection with a short left-turn bay is closely related to these factors, it is very important to more precisely calculate the probability of the occurrence of left-turn bay blockage and spillback situations.

In order to calculate these probabilities, Matlab program was coded to conduct the following procedures. Firstly, the probability that each number of left-turning and through vehicles arrive at the intersection during a designated time should be computed using the following equation:

1. The probability density function for counts of through vehicles following a binomial distribution is as follow:

$$P_{TH}(i) = \frac{\hat{n}!}{i!(\hat{n}-i)!}\hat{p}^{i}(1-\hat{p})^{n-i} \tag{6}$$

where,

n = the total number of through cars (from the Eq. (4))

i = the number of the through vehicles arriving at the intersection

$P_{TH}(i)$ = the probability with i number of through vehicles arriving (from the Eq. (3))

2. The probability density function for counts of left-turn vehicles following a Poisson distribution is as follow:

$$P_{LT}(j) = \frac{e^{-m} \times m^{j}}{j!} \tag{7}$$

where,

m = the average number of vehicles arriving at the intersection

j = the number of vehicles arriving at the intersection during a designated time

Secondly, the probability about each case that left-turning and through cars randomly occurs needs to be determined. That is, the required values for this calculation are the ratio of each case to all possible cases and the probability that the number of left-turning and through vehicles included in each case arrives at the intersection. So, the probability for each case can be determined by multiplying all these values because the arrival of left-turn and through cars are independent of each other. Here, it is needed to judge if each case is included in the blockage situation or spillback condition by defining conditions for becoming the left-turn bay blockage and spillback. Finally, the probability of left-turn bay blockage and spillback can be calculated using the following equations by adding the probabilities of all cases that satisfies the left-turn bay blockage and spillback situation, respectively.

1. The probability of left-turn bay blockage (P_{block}) :

$$P_{block}(k) = \sum_{j=m_L}^{n_L} \sum_{i=m_T}^{n_T} \frac{N_{B(i,j,k)}}{N_{Total(i,j)}} \cdot P_{TH}(i) \cdot P_{LT}(j) \tag{8}$$

where,

k = The length of left-turn bay (number of vehicles)

m_T = The minimum value in the realistic range for through vehicles related to the left-turn blockage situation

n_T = The maximum value in the realistic range for through vehicles related to the left-turn blockage situation

m_L = The minimum value in the realistic range for left-turn vehicles related to the left-turn blockage situation

n_L = The maximum value in the realistic range for left-turn vehicles related to the left-turn blockage situation

$N_{B(i,j,k)}$ = The number of the left-turn bay blockages cases when the length of the left-turn bay is k with i through vehicles and j left-turn cars

$N_{Total(i,j)}$ = The total number of all cases that can occur with i through vehicles and j left-turn cars

2. The probability of left-turn bay spillback (P_{spill}) :

$$P_{Spill}(k) = \sum_{i=m_T}^{n_T} \sum_{j=m_L}^{n_L} \frac{N_{S(i,j,k)}}{N_{Total(i,j)}} \cdot P_{TH}(i) \cdot P_{LT}(j) \tag{9}$$

where,

k = The length of left-turn bay (number of vehicles)

m_T = The minimum value in the realistic range for through vehicles related to the left-turn spillback situation

n_T = The maximum value in the realistic range for through vehicles related to the left-turn spillback situation

m_L = The minimum length of left-turn bay + 2

n_L = The maximum value in the realistic range for left-turn vehicles related to the left-turn spillback situation

$N_{S(i,j,k)}$ = The number of the left-turn bay spillback cases when the length of the left-turn bay is *k* with *i* through vehicles and *j* left-turn cars

$N_{Total(i,j)}$ = The total number of all cases that can occur with *i* through vehicles and *j* left-turn cars

The conditions defined as the blockage and spillback are as follow:

1) Definition for the Left-Turn Bay Blockage situation:

The left-turn bay containing the transitional area commonly accomodates 0 to (*k*+2) left-turn vehicles, and there are two cases of left-turn bay blockage situations (see Figure 7).

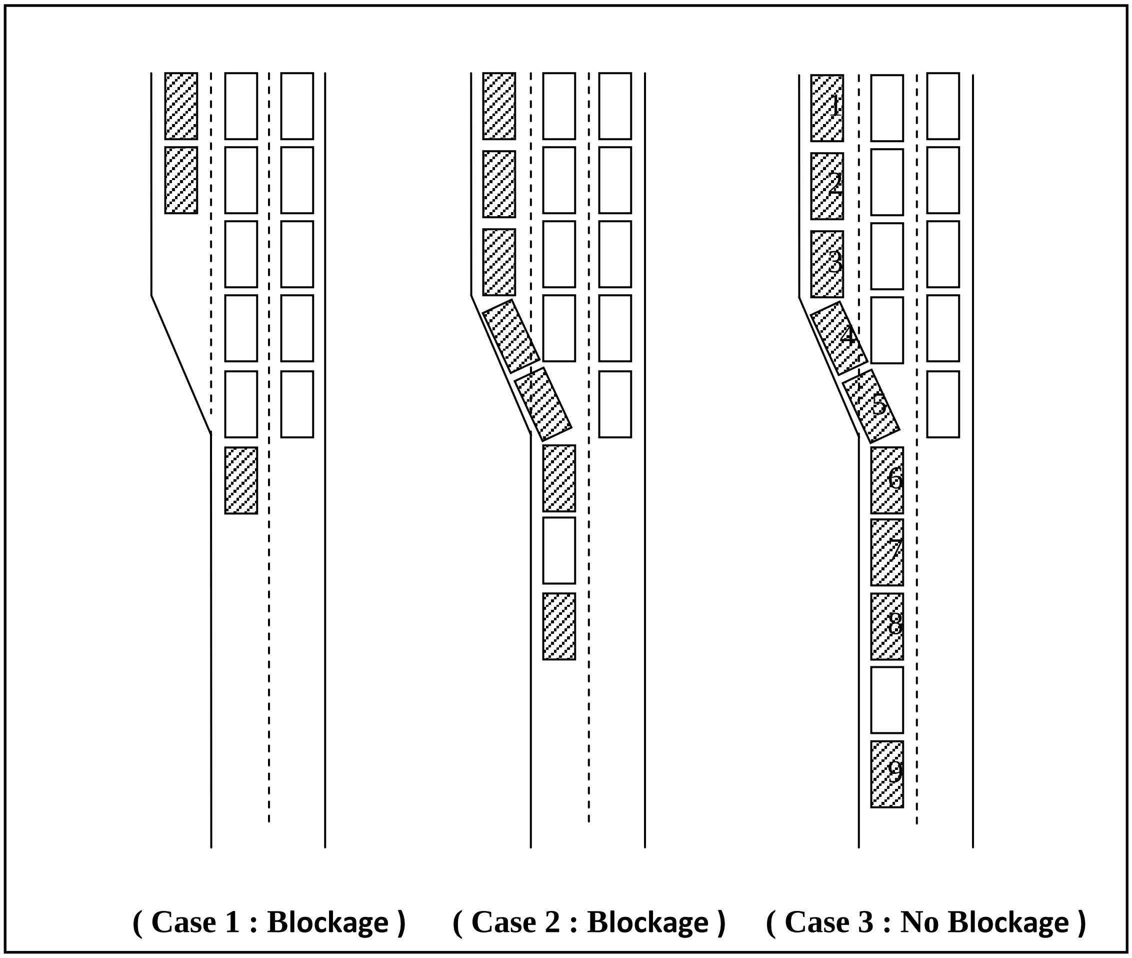

FIGURE 7 Several cases related to the left-turn bay blockage situation

As shown by Case 1 in Figure 7, the moment *k*+2 through vehicles arrive at the intersection in the adjacent through lane when there are no left-turn bay spillbacks during the red interval, the left-turn bay blockage situation will take place (Zhang and Tong, 2007). Also, Case 2, shown in Figure 7, should be included in the left-turn bay blockage situation even though there are left-turn bay spillbacks during the red interval. This is because all left-turn cars in the left-turn bay spillback can pass during the protected LT green time, and then the following through vehicle blocks the left-turn vehicles behind it while the protected LT green time is still displayed. That is, the left-turn bay blockage situations should include cases when the number of left-turn vehicles forming the spillback is smaller than the maximum left-turn cars that can pass during the protected LT green time and there are through vehicles in the queue that eventually will block the entrance to the left-turn bay.

In this research, the maximum number of left-turn vehicles that can be discharged during the protected LT green time (17 seconds) is 8 vehicles, given the green time and saturation headway. This number can be obtained from the following equation (Carroll J. Messer and Daniel B. Fambro, 1977):

$$T_c = 2.0 + 2.0N_p \tag{10}$$

where,

T_c = Time after start of green for the automobile in queue storage position number N_p to clear the stop line on the approach (Seconds)

N_p = Queue storage position number for left-turning automobile (Here, the N_p of the first left-turn vehicle is 0)

Like Case 3 in Figure 7, when the number of left-turn vehicles in the spillback is more than 7 vehicles, this situation can not be a left-turn bay blockage situation because the 9th left-turn vehicle is not able to pass regardless of the blockage condition by the adjacent through vehicles.

Also, since most left-turn bay blockage situations occur during the red interval, only the values of mean and variance for through and left-turn vehicles arriving during this time are considered. Here, the realistic and specific range in Equation 8 needs to be reasonably defined. That is, the range for the through movement was given as $\bar{x} \pm 2.5s$, and the left-turn vehicles were considered from 0 to $2\times\bar{x}$ vehicles. Beyond this range, the probability is so small and can be ignored in the calculation using Equation 8. Sample values are calculated through data obtained from the CORSIM and provided in Table 16:

TABLE 16 Mean and variance of vehicles related to the left-turn bay blockage

	Through vehicles	Left-turn vehicles
Mean ($\bar{x}$)	21.15	4.01
Variance (s^2)	8.72	6.05
Range	13 ~ 29	0 ~ 8

It was assumed that the blockage situation happens with the arrival of 2(k+2) through vehicles because Matlab program cannot discriminate vehicles in the two through lanes. Therefore, the probability of left-turn bay blockage by through traffic is produced by the following equation:

$$P_{block} = P\{(X_{TH} \geq 2k + 4) \cap (X_{LT} \leq N_m - 1)\}$$

where,

N_m = Left-turn phase capacity, or the maximum number left-turn vehicles that can be discharged during the protected LT green time

Additionally, it is necessary to keep in mind that this left-turn bay blockage situation only occurs when at least one left-turning vehicle exists during the protected LT green time after the blockage situation occurs.

2) Definition for the Left-Turn Bay Spillback situation:

The left-turn bay spillback situation is defined as having at least k+2 left-turn cars arrived at the intersection during the green interval for through movement after the protected left-turn phase ends. Since most left-turn bay spillback situation at a signalized intersection with a short left-turn bay occurs during this interval, only the value of mean and variance for through and left-turn vehicles arriving during this time are used to define the spillback probability. Also, the appropriate range for each movement should be specified due to irregular arrivals in the real-world. As stated above in the blockage situation, the range for the through movement was given as $\bar{x} \pm 2.5s$, and the left-turn vehicles were considered from 0 to $2 \times \bar{x}$ vehicles. Sample values are provided in Table 17:

TABLE 17 Mean and variance of vehicles related to the left-turn bay spillback

	Through vehicles	Left-turn vehicles
Mean	23.65	4.58
Variance	8.56	5.33
Range	16 ~ 31	0 ~ 9

So, the probability of left-turn bay spillback is generated by the following simple equation:

$$P_{spill} = P(X_{LT} \geq k + 2)$$

Here, this left-turn bay spillback situation only takes place when at least one through vehicle exists during the TH green time after the spillback situation occurs.

Furthermore, in order to more precisely calculate the probability of the left-turn spillback situation at a signalized intersection with short left-turn bay under the leading left-turn signal, the left-turn blockage situation should be considered. This is because the left-turn blockage situation will produce residue queue that lead to spillback situation. Therefore, it is required to compute the probability of the left-turn bay spillback by considering the following two scenarios:

1. The probability that the left-turn bay spillback situation occurs without the occurrence of the left-turn bay blockage situation ($P_{(spill,\ no\ block)}$) : This probability can be obtained using Eq. (9).

2. The probability that the left-turn bay spillback situation occurs after the occurrence of the left-turn bay blockage situation ($P_{(spill,\ block)}$): This probability can be also calculated using Eq. (9). However, the number of left-turn vehicles remaining on the adjacent through lane at the end of the protected LT green time due to blockages should be subtracted from the value of m_L. This subtraction is necessary because, depending on the number of remaining left-turn cars on the adjacent through lane, fewer left-turn vehicles are required in order for the left-turn bay spillback situation to occur.

In the end, the probability of the left-turn bay spillback situation, considering the above two scenarios, can be calculated using the following equation:

$$P_{spill} = (1\text{-}P_{block}) \times P_{(spill,\ no\ block)} + P_{block} \times P_{(spill,\ block)} \qquad (11)$$

Firstly, the values of all variables required to get the probability of the left-turn bay blockage situation, P_{block}, using Eq. (8) are provided in Tables 18 and 19. For comparison purpose, the P_{block} is calculated considering both the Poisson distribution as well as the Binomial distribution for the through movement in a heavy traffic. Both distributions are also considered for the through traffic when the probability of the left-turn bay spillback, P_{spill}, is computed. Table 20 presents the probability that the left-turn bay blockage situation takes place under heavy traffic condition according to the length of left-turn bay.

TABLE 18 The value of all variables required to get the P_{block}

TH (i)	LT (j)	$N_{Total(i,j)}$	Binomial $P_{TH}(i)\cdot P_{LT}(j)$	Poisson $P_{TH}(i)\cdot P_{LT}(j)$	LT (j)	$N_{Total(i,j)}$	Binomial $P_{TH}(i)\cdot P_{LT}(j)$	Poisson $P_{TH}(i)\cdot P_{LT}(j)$
13	1	14	0.0002	0.0013	3	560	0.0006	0.0035
14		15	0.0006	0.0020		680	0.0015	0.0052
15		16	0.0012	0.0028		816	0.0031	0.0074
16		17	0.0022	0.0036		969	0.0058	0.0097
17		18	0.0036	0.0045		1140	0.0098	0.0121
18		19	0.0055	0.0053		1330	0.0147	0.0142
19		20	0.0074	0.0059		1540	0.0198	0.0159
20		21	0.0090	0.0063		1771	0.0240	0.0168
21		22	0.0097	0.0063		2024	0.0261	0.0169
22		23	0.0094	0.0061		2300	0.0253	0.0162
23		24	0.0082	0.0056		2600	0.0219	0.0149
24		25	0.0063	0.0049		2925	0.0169	0.0132
25		26	0.0043	0.0042		3276	0.0116	0.0111
26		27	0.0026	0.0034		3654	0.0070	0.0091
27		28	0.0014	0.0026		4060	0.0037	0.0071
28		29	0.0006	0.0020		4495	0.0017	0.0054
29		30	0.0002	0.0015		4960	0.0007	0.0039
13	2	105	0.0005	0.0026	4	2380	0.0006	0.0035
14		120	0.0011	0.0039		3060	0.0015	0.0052
15		136	0.0023	0.0055		3876	0.0031	0.0074
16		153	0.0044	0.0073		4845	0.0059	0.0098
17		171	0.0073	0.0091		5985	0.0098	0.0122
18		190	0.0110	0.0107		7315	0.0147	0.0143
19		210	0.0148	0.0119		8855	0.0199	0.0159
20		231	0.0180	0.0125		10626	0.0241	0.0168
21		253	0.0195	0.0126		12650	0.0261	0.0169
22		276	0.0189	0.0121		14950	0.0254	0.0163
23		300	0.0164	0.0112		17550	0.0220	0.0150
24		325	0.0127	0.0098		20475	0.0170	0.0132
25		351	0.0087	0.0083		23751	0.0116	0.0112
26		378	0.0052	0.0068		27405	0.0070	0.0091
27		406	0.0028	0.0053		31465	0.0037	0.0071
28		435	0.0013	0.0040		35960	0.0017	0.0054
29		465	0.0005	0.0029		40920	0.0007	0.0039

TABLE 18 Continued

TH (*i*)	LT (*j*)	$N_{Total(i,j)}$	Binomial $P_{TH}(i)\cdot P_{LT}(j)$	Poisson $P_{TH}(i)\cdot P_{LT}(j)$	LT (*j*)	$N_{Total(i,j)}$	Binomial $P_{TH}(i)\cdot P_{LT}(j)$	Poisson $P_{TH}(i)\cdot P_{LT}(j)$
13	5	8568	0.0005	0.0028	7	77520	0.0002	0.0011
14		11628	0.0012	0.0042		116280	0.0005	0.0016
15		15504	0.0025	0.0059		170544	0.0010	0.0023
16		20349	0.0047	0.0078		245157	0.0018	0.0030
17		26334	0.0079	0.0097		346104	0.0030	0.0037
18		33649	0.0118	0.0115		480700	0.0045	0.0044
19		42504	0.0159	0.0127		657800	0.0061	0.0049
20		53130	0.0193	0.0135		888030	0.0074	0.0052
21		65780	0.0210	0.0136		1184040	0.0080	0.0052
22		80730	0.0203	0.0131		1560780	0.0078	0.0050
23		98280	0.0176	0.0120		2035800	0.0068	0.0046
24		118755	0.0136	0.0106		2629575	0.0052	0.0041
25		142506	0.0093	0.0089		3365856	0.0036	0.0034
26		169911	0.0056	0.0073		4272048	0.0021	0.0028
27		201376	0.0030	0.0057		5379616	0.0011	0.0022
28		237336	0.0014	0.0043		6724520	0.0005	0.0016
29		278256	0.0005	0.0031		8347680	0.0002	0.0012
13	6	27132	0.0003	0.0019	8	203490	0.0001	0.0005
14		38760	0.0008	0.0028		319770	0.0002	0.0008
15		54264	0.0017	0.0040		490314	0.0005	0.0011
16		74613	0.0031	0.0052		735471	0.0009	0.0015
17		100947	0.0053	0.0065		1081575	0.0015	0.0019
18		134596	0.0079	0.0077		1562275	0.0023	0.0022
19		177100	0.0107	0.0085		2220075	0.0031	0.0024
20		230230	0.0129	0.0090		3108105	0.0037	0.0026
21		296010	0.0140	0.0091		4292145	0.0040	0.0026
22		376740	0.0136	0.0087		5852925	0.0039	0.0025
23		475020	0.0118	0.0080		7888725	0.0034	0.0023
24		593775	0.0091	0.0071		10518300	0.0026	0.0020
25		736281	0.0062	0.0060		13884156	0.0018	0.0017
26		906192	0.0037	0.0049		18156204	0.0011	0.0014
27		1107568	0.0020	0.0038		23535820	0.0006	0.0011
28		1344904	0.0009	0.0029		30260340	0.0003	0.0008
29		1623160	0.0004	0.0021		38608020	0.0001	0.0006

Table 18 presents the total number ($N_{Total(i,j)}$) of all cases that can occur with i through vehicles and j left-turn cars and the probability that these through and left-turn cars arrive at the intersection during a designated time. As shown in Table 18, this probability was calculated using the binomial and the Poisson distribution, respectively. Here, it can be seen that the probability from the binomial distribution tends to be higher than the Poisson distribution as the through and left-turn vehicles get closer and closer to the value of mean. That is, this fact reminds us that the binomial distribution should be a better estimator as variance of traffic data in the real-world is getting smaller and smaller compared to the value of mean.

Also, the value of these variables, indicated in Table 18, is constant irrespective of the length of the left-turn bay because these variables are only affected by the number of through and left-turn vehicles. So, it is essential to get the number of the left-turn bay blockages cases, $N_{B(i,j,k)}$, when the length of the left-turn bay is k with i through vehicles and j left-turn cars because this variable substantially determines the difference of the P_{block} according to the different length of left-turn bay. The values of $N_{B(i,j,k)}$ were calculated from the Matlab and indicated in Table 19, and this code was shown in the APPENDIX C.

TABLE 19 The number of the left-turn bay blockage cases according to the length of left-turn bay

TH (i)	LT (j)	$N_{B(i,j,3)}$	$N_{B(i,j,4)}$	$N_{B(i,j,5)}$	$N_{B(i,j,6)}$	$N_{B(i,j,7)}$	$N_{B(i,j,8)}$	$N_{B(i,j,9)}$	$N_{B(i,j,10)}$
13	1	4	2	0	0	0	0	0	0
14		5	3	1	0	0	0	0	0
15		6	4	2	0	0	0	0	0
16		7	5	3	1	0	0	0	0
17		8	6	4	2	0	0	0	0
18		9	7	5	3	1	0	0	0
19		10	8	6	4	2	0	0	0
20		11	9	7	5	3	1	0	0
21		12	10	8	6	4	2	0	0
22		13	11	9	7	5	3	1	0
23		14	12	10	8	6	4	2	0
24		15	13	11	9	7	5	3	1
25		16	14	12	10	8	6	4	2
26		17	15	13	11	9	7	5	3
27		18	16	14	12	10	8	6	4
28		19	17	15	13	11	9	7	5
29		20	18	16	14	12	10	8	6
13	2	50	27	0	0	0	0	0	0
14		65	42	15	0	0	0	0	0
15		81	58	31	0	0	0	0	0
16		98	75	48	17	0	0	0	0
17		116	93	66	35	0	0	0	0
18		135	112	85	54	19	0	0	0
19		155	132	105	74	39	0	0	0
20		176	153	126	95	60	21	0	0
21		198	175	148	117	82	43	0	0
22		221	198	171	140	105	66	23	0
23		245	222	195	164	129	90	47	0
24		270	247	220	189	154	115	72	25
25		296	273	246	215	180	141	98	51
26		323	300	273	242	207	168	125	78
27		351	328	301	270	235	196	153	106
28		380	357	330	299	264	225	182	135
29		410	387	360	329	294	255	212	165

TABLE 19 Continued.

TH (*i*)	LT (*j*)	$N_{B(i,j,3)}$	$N_{B(i,j,4)}$	$N_{B(i,j,5)}$	$N_{B(i,j,6)}$	$N_{B(i,j,7)}$	$N_{B(i,j,8)}$	$N_{B(i,j,9)}$	$N_{B(i,j,10)}$
13		340	196	0	0	0	0	0	0
14		460	316	120	0	0	0	0	0
15		596	452	256	0	0	0	0	0
16		749	605	409	153	0	0	0	0
17		920	776	580	324	0	0	0	0
18		1110	966	770	514	190	0	0	0
19		1320	1176	980	724	400	0	0	0
20		1551	1407	1211	955	631	231	0	0
21	**3**	1804	1660	1464	1208	884	484	0	0
22		2080	1936	1740	1484	1160	760	276	0
23		2380	2236	2040	1784	1460	1060	576	0
24		2705	2561	2365	2109	1785	1385	901	325
25		3056	2912	2716	2460	2136	1736	1252	676
26		3434	3290	3094	2838	2514	2114	1630	1054
27		3840	3696	3500	3244	2920	2520	2036	1460
28		4275	4131	3935	3679	3355	2955	2471	1895
29		4740	4596	4400	4144	3820	3420	2936	2360
13		1665	1015	0	0	0	0	0	0
14		2345	1695	680	0	0	0	0	0
15		3161	2511	1496	0	0	0	0	0
16		4130	3480	2465	969	0	0	0	0
17		5270	4620	3605	2109	0	0	0	0
18		6600	5950	4935	3439	1330	0	0	0
19		8140	7490	6475	4979	2870	0	0	0
20		9911	9261	8246	6750	4641	1771	0	0
21	**4**	11935	11285	10270	8774	6665	3795	0	0
22		14235	13585	12570	11074	8965	6095	2300	0
23		16835	16185	15170	13674	11565	8695	4900	0
24		19760	19110	18095	16599	14490	11620	7825	2925
25		23036	22386	21371	19875	17766	14896	11101	6201
26		26690	26040	25025	23529	21420	18550	14755	9855
27		30750	30100	29085	27589	25480	22610	18815	13915
28		35245	34595	33580	32084	29975	27105	23310	18410
29		40205	39555	38540	37044	34935	32065	28270	23370
13		6566	4200	0	0	0	0	0	0
14		9626	7260	3060	0	0	0	0	0
15		13502	11136	6936	0	0	0	0	0

TABLE 19 Continued.

TH (*i*)	LT (*j*)	$N_{B(i,j,3)}$	$N_{B(i,j,4)}$	$N_{B(i,j,5)}$	$N_{B(i,j,6)}$	$N_{B(i,j,7)}$	$N_{B(i,j,8)}$	$N_{B(i,j,9)}$	$N_{B(i,j,10)}$
16	**5**	18347	15981	11781	4845	0	0	0	0
17		24332	21966	17766	10830	0	0	0	0
18		31647	29281	25081	18145	7315	0	0	0
19		40502	38136	33936	27000	16170	0	0	0
20		51128	48762	44562	37626	26796	10626	0	0
21		63778	61412	57212	50276	39446	23276	0	0
22		78728	76362	72162	65226	54396	38226	14950	0
23		96278	93912	89712	82776	71946	55776	32500	0
24		116753	114387	110187	103251	92421	76251	52975	20475
25		140504	138138	133938	127002	116172	100002	76726	44226
26		167909	165543	161343	154407	143577	127407	104131	71631
27		199374	197008	192808	185872	175042	158872	135596	103096
28		235334	232968	228768	221832	211002	194832	171556	139056
29		276254	273888	269688	262752	251922	235752	212476	179976
13	**6**	22127	14756	0	0	0	0	0	0
14		33755	26384	11628	0	0	0	0	0
15		49259	41888	27132	0	0	0	0	0
16		69608	62237	47481	20349	0	0	0	0
17		95942	88571	73815	46683	0	0	0	0
18		129591	122220	107464	80332	33649	0	0	0
19		172095	164724	149968	122836	76153	0	0	0
20		225225	217854	203098	175966	129283	53130	0	0
21		291005	283634	268878	241746	195063	118910	0	0
22		371735	364364	349608	322476	275793	199640	80730	0
23		470015	462644	447888	420756	374073	297920	179010	0
24		588770	581399	566643	539511	492828	416675	297765	118755
25		731276	723905	709149	682017	635334	559181	440271	261261
26		901187	893816	879060	851928	805245	729092	610182	431172
27		1102563	1095192	1080436	1053304	1006621	930468	811558	632548
28		1339899	1332528	1317772	1290640	1243957	1167804	1048894	869884
29		1618155	1610784	1596028	1568896	1522213	1446060	1327150	1148140
13	**7**	66080	45696	0	0	0	0	0	0
14		104840	84456	38760	0	0	0	0	0
15		159104	138720	93024	0	0	0	0	0
16		233717	213333	167637	74613	0	0	0	0
17		334664	314280	268584	175560	0	0	0	0
18		469260	448876	403180	310156	134596	0	0	0

TABLE 19 Continued.

TH (*i*)	LT (*j*)	$N_{B(i,j,3)}$	$N_{B(i,j,4)}$	$N_{B(i,j,5)}$	$N_{B(i,j,6)}$	$N_{B(i,j,7)}$	$N_{B(i,j,8)}$	$N_{B(i,j,9)}$	$N_{B(i,j,10)}$
19		646360	625976	580280	487256	311696	0	0	0
20		876590	856206	810510	717486	541926	230230	0	0
21		1172600	1152216	1106520	1013496	837936	526240	0	0
22		1549340	1528956	1483260	1390236	1214676	902980	376740	0
23		2024360	2003976	1958280	1865256	1689696	1378000	851760	0
24	**7**	2618135	2597751	2552055	2459031	2283471	1971775	1445535	593775
25		3354416	3334032	3288336	3195312	3019752	2708056	2181816	1330056
26		4260608	4240224	4194528	4101504	3925944	3614248	3088008	2236248
27		5368176	5347792	5302096	5209072	5033512	4721816	4195576	3343816
28		6713080	6692696	6647000	6553976	6378416	6066720	5540480	4688720
29		8336240	8315856	8270160	8177136	8001576	7689880	7163640	6311880
13		179180	127908	0	0	0	0	0	0
14		295460	244188	116280	0	0	0	0	0
15		466004	414732	286824	0	0	0	0	0
16		711161	659889	531981	245157	0	0	0	0
17		1057265	1005993	878085	591261	0	0	0	0
18		1537965	1486693	1358785	1071961	480700	0	0	0
19		2195765	2144493	2016585	1729761	1138500	0	0	0
20		3083795	3032523	2904615	2617791	2026530	888030	0	0
21	**8**	4267835	4216563	4088655	3801831	3210570	2072070	0	0
22		5828615	5777343	5649435	5362611	4771350	3632850	1558651	0
23		7864415	7813143	7685235	7398411	6807150	5668650	3592322	0
24		10493990	10442718	10314810	10027986	9436725	8298225	6219768	2607093
25		13859846	13808574	13680666	13393842	12802581	11664081	9583495	5950467
26		18131894	18080622	17952714	17665890	17074629	15936129	13853414	10200033
27		23511510	23460238	23332330	23045506	22454245	21315745	19230901	15557167
28		30236030	30184758	30056850	29770026	29178765	28040265	25953292	22259205
29		38583710	38532438	38404530	38117706	37526445	36387945	34298843	30584403

As it is expected, the value of $N_{B(i,j,k)}$ decreases as the length of left-turn bay increases. Also, it is seen that the value of $N_{B(i,j,k)}$ definitely increases as the number of through and left-turn vehicles considered is getting more and more. As shown in Table

19, the $N_{B(i,j,k)}$ is equal to zero when the number of through vehicles, i, arriving at the intersection is smaller than $2k+4$ vehicles because this case can not cause any left-turn bay blockage situations. In other words, the left-turn bay blockage situation can occur only when more than $2k+4$ through vehicles arrive at the intersection. So, as the length of left-turn bay increases, the $N_{B(i,j,k)}$ should decreases because more through vehicles are required to cause a blockage situation.

By using all these values shown in Table 18 and 19, the probability of the left-turn bay blockage situation, P_{block}, according to the length of left-turn bay can be obtained using Eq. (8) and are presented in Table 20 and Figure 8:

TABLE 20 Summary of the P_{block} according to the length of left-turn bay

Distribution	P_{block} (3)	P_{block} (4)	P_{block} (5)	P_{block} (6)	P_{block} (7)	P_{block} (8)	P_{block} (9)	P_{block} (10)
Binomial	0.8459	0.7865	0.7012	0.5835	0.4350	0.2747	0.1376	0.0509
Poisson	0.7850	0.7209	0.6270	0.5082	0.4006	0.2536	0.1501	0.0757

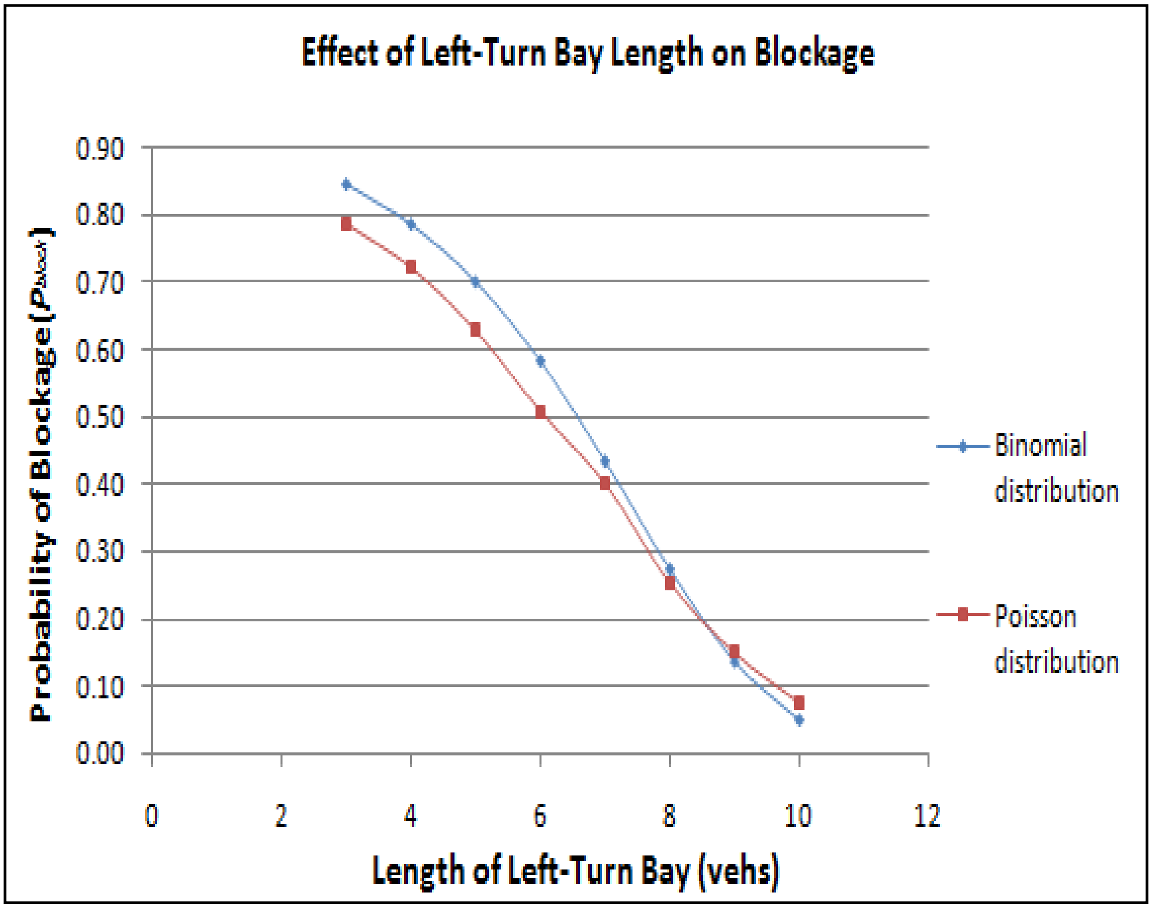

FIGURE 8 Effect of left-turn bay length on the probability of blockage

It can be obviously seen that the P_{block} is dwindling as the length of left-turn bay increases. This means that interruption of the left-turn traffic flows due to the left-turn bay blockage situation is getting worse and worse as the length of the left-turn lane is reduced. The maximum reduction of the P_{block} in both distributions was indicated when the length of the left-turn bay changed from 7 to 8, the values of the maximum reduction in the binomial and Poisson distribution are 0.1601 and 0.1471, respectively.

Also, on average, the P_{block} of the binomial distribution indicated to be 3.68% higher than the P_{block} of the Poisson distribution. The P_{block} of the binomial distribution is also showed to decrease sharper than the P_{block} of the Poisson distribution. As the length of the left-turning bay increases, the difference in blockage probability from the two distributions becomes smaller and eventfully the blockage probability from the Poisson distribution becomes slightly higher than that of the binomial distribution.

In addition, whenever a blockage situation takes place, the average number, of the left-turn vehicles that can pass the intersection considering this blockage situation, $E(X_{LT})$, can be obtained using the following equation :

$$E(X_{LT}) = \sum_{x=m_L+1}^{n_L-1} xf(x) \tag{12}$$

Here, x is defined as the number of left-turn vehicles passing the intersection before the occurrence of a blockage situation, and $f(x)$ is the probability that left-turn bay blockage situations occur with x number of left-turn vehicles passing the intersection when a given number of left-turn and through vehicles arrive at the intersection. The probability that x number of left-turn vehicles can pass the intersection under the left-turn bay blockage situation can be calculated using the following equation:

$$f(x) = \sum_{j=m_L+2}^{n_L} \sum_{i=m_T}^{n_T} \frac{N_{B(i,x,k)}}{N_{Total(i,j)}} \cdot P_{TH}(i) \cdot P_{LT}(j) \tag{13}$$

In this equation, $N_{B(i,x,k)}$ is the number of the left-turn bay blockages cases with x number of left-turn vehicles passing the intersection when the length of the left-turn bay is *k*, and *i* through vehicles and *j* left-turn cars arrive at the intersection. Also, since "x = 0" does not affect the value in Eq.(12), at least 2 left-turn vehicles should be arriving at the intersection. This is because the value of *x* would be zero if a left-turn bay blockage situation takes place with only one left-turn vehicle comes to the intersection. Through Eq. (12) and Eq. (13), the values of $E(X_{LT})$ according to the length of left-turn bay are summarized in Table 21:

TABLE 21 Summary of the $E(X_{LT})$ according to the length of left-turn bay

Distribution	**The length of left-turn bay**							
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Binomial	1.43	1.55	1.56	1.41	1.12	0.74	0.43	0.17
Poisson	1.35	1.43	1.39	1.21	0.96	0.67	0.47	0.21

Next, the probability of the left-turn bay spillback, P_{spill}, needs to be calculated using Eq. (9) and Eq. (11). The value of all variables required to calculate the probability of the left-turn bay spillback, P_{spill}, using Eq. (9) are provided in Tables 22 ~ 25 on pages 68, 71, and 73, and the probability (P_{spill}) that the left-turn bay spillback situation takes place under heavy traffic condition according to the length of left-turn bay is presented in Table 26:

TABLE 22 The value of all variables required to get the P_{spill}

TH (*i*)	LT (*j*)	$N_{Total(i,j)}$	Binomial $P_{TH}(i) \cdot P_{LT}(j)$	Poisson $P_{TH}(i) \cdot P_{LT}(j)$	TH (*i*)	LT (*j*)	$N_{Total(i,j)}$	Binomial $P_{TH}(i) \cdot P_{LT}(j)$	Poisson $P_{TH}(i) \cdot P_{LT}(j)$
16	2	153	0.0008	0.0026	21	2	253	0.0094	0.0080
	3	969	0.0012	0.0040		3	2024	0.0144	0.0122
	4	4845	0.0013	0.0046		4	12650	0.0165	0.0140
	5	20349	0.0012	0.0042		5	65780	0.0151	0.0128
	6	74613	0.0009	0.0032		6	296010	0.0115	0.0098
	7	245157	0.0006	0.0021		7	1184040	0.0075	0.0064
	8	735471	0.0004	0.0012		8	4292145	0.0043	0.0037
	9	2042975	0.0002	0.0006		9	14307150	0.0022	0.0019
17	2	171	0.0015	0.0037	22	2	276	0.0117	0.0086
	3	1140	0.0023	0.0056		3	2300	0.0178	0.0131
	4	5985	0.0027	0.0064		4	14950	0.0204	0.0150
	5	26334	0.0025	0.0059		5	80730	0.0187	0.0138
	6	100947	0.0019	0.0045		6	376740	0.0143	0.0105
	7	346104	0.0012	0.0029		7	1560780	0.0093	0.0069
	8	1081575	0.0007	0.0017		8	5852925	0.0053	0.0039
	9	3124550	0.0004	0.0009		9	20160075	0.0027	0.0020
18	2	190	0.0028	0.0048	23	2	300	0.0131	0.0088
	3	1330	0.0043	0.0074		3	2600	0.0201	0.0135
	4	7315	0.0049	0.0084		4	17550	0.0230	0.0154
	5	33649	0.0045	0.0077		5	98280	0.0210	0.0141
	6	134596	0.0034	0.0059		6	475020	0.0161	0.0108
	7	480700	0.0022	0.0039		7	2035800	0.0105	0.0071
	8	1562275	0.0013	0.0022		8	7888725	0.0060	0.0040
	9	4686825	0.0007	0.0011		9	28048800	0.0031	0.0021
19	2	210	0.0046	0.0060	24	2	325	0.0134	0.0087
	3	1540	0.0070	0.0092		3	2925	0.0205	0.0133
	4	8855	0.0081	0.0105		4	20475	0.0235	0.0152
	5	42504	0.0074	0.0096		5	118755	0.0215	0.0139
	6	177100	0.0056	0.0073		6	593775	0.0164	0.0106
	7	657800	0.0037	0.0048		7	2629575	0.0107	0.0070
	8	2220075	0.0021	0.0027		8	10518300	0.0062	0.0040
	9	6906900	0.0011	0.0014		9	38567100	0.0031	0.0020
20	2	231	0.0069	0.0071	25	2	351	0.0125	0.0082

TABLE 22 Continued.

TH (*i*)	LT (*j*)	$N_{Total(i,j)}$	Binomial $P_{TH}(i)\cdot P_{LT}(j)$	Poisson $P_{TH}(i)\cdot P_{LT}(j)$	TH (*i*)	LT (*j*)	$N_{Total(i,j)}$	Binomial $P_{TH}(i)\cdot P_{LT}(j)$	Poisson $P_{TH}(i)\cdot P_{LT}(j)$
20	**3**	1771	0.0105	0.0108	**25**	**3**	3276	0.0190	0.0126
	4	10626	0.0121	0.0124		**4**	23751	0.0218	0.0144
	5	53130	0.0111	0.0114		**5**	142506	0.0199	0.0132
	6	230230	0.0084	0.0087		**6**	736281	0.0152	0.0101
	7	888030	0.0055	0.0057		**7**	3365856	0.0100	0.0066
	8	3108105	0.0032	0.0032		**8**	13884156	0.0057	0.0038
	9	10015005	0.0016	0.0017		**9**	52451256	0.0029	0.0019
26	**2**	378	0.0104	0.0075	**29**	**2**	465	0.0033	0.0045
	3	3654	0.0159	0.0114		**3**	4960	0.0050	0.0069
	4	27405	0.0183	0.0131		**4**	40920	0.0058	0.0079
	5	169911	0.0167	0.0120		**5**	278256	0.0053	0.0072
	6	906192	0.0128	0.0092		**6**	1623160	0.0040	0.0055
	7	4272048	0.0084	0.0060		**7**	8347680	0.0026	0.0036
	8	18156204	0.0048	0.0034		**8**	38608020	0.0015	0.0021
	9	70607460	0.0024	0.0017		**9**	163011640	0.0008	0.0011
27	**2**	406	0.0079	0.0066	**30**	**2**	496	0.0018	0.0036
	3	4060	0.0121	0.0100		**3**	5456	0.0027	0.0054
	4	31465	0.0138	0.0115		**4**	46376	0.0031	0.0062
	5	201376	0.0127	0.0105		**5**	324632	0.0029	0.0057
	6	1107568	0.0097	0.0080		**6**	1947792	0.0022	0.0044
	7	5379616	0.0063	0.0052		**7**	10295472	0.0014	0.0028
	8	23535820	0.0036	0.0030		**8**	48903492	0.0008	0.0016
	9	94143280	0.0018	0.0015		**9**	211915132	0.0004	0.0008
28	**2**	435	0.0054	0.0055	**31**	**2**	528	0.0009	0.0027
	3	4495	0.0082	0.0085		**3**	5984	0.0013	0.0042
	4	35960	0.0094	0.0097		**4**	52360	0.0015	0.0048
	5	237336	0.0086	0.0089		**5**	376992	0.0014	0.0044
	6	1344904	0.0066	0.0068		**6**	2324784	0.0011	0.0033
	7	6724520	0.0043	0.0044		**7**	12620256	0.0007	0.0022
	8	30260340	0.0025	0.0025		**8**	61523748	0.0004	0.0012
	9	124403620	0.0013	0.0013		**9**	273438880	0.0002	0.0006

Similarly to the left-turn bay blockage situation, Table 22 indicated the total number ($N_{Total(i,j)}$) of all cases that can occur with i through vehicles and j left-turn cars and the probability that these number of through and left-turn cars arrive at the intersection during a designated time. This probability was also calculated using the binomial and the Poisson distribution, and the probability from the binomial distribution tends to be higher than the Poisson distribution as each number of through and left-turn vehicles is getting closer and closer to the value of mean. Also, the value of these values, shown in Table 22, doesn't change with respect to the length of left-turn bay because these variables are only affected by the number of through and left-turn vehicles. So, the number of the left-turn bay spillback cases, $N_{S(i,j,k)}$ is calculated the same as the $N_{B(i,j,k)}$ of the left-turn bay blockage situation. Here, in order to use Eq. (11), the $N_{S(i,j,k)}$ should be obtained by considering two scenarios. The first scenario is that the left-turn bay spillback situations occur when no previous left-turn bay blockage situation exists. The other scenario is that the left-turn bay spillback situations take place with a previous left-turn bay blockage situation. In order to get the value of $N_{S(i,j,k)}$ regarding the second scenario, it is required to estimate how many left-turn vehicles, on average, remained in the adjacent through lane after the occurrence of blockage situation. These values were obtained by subtracting $E(X_{LT})$, shown in Table 21, from the mean of left-turn vehicles related to the left-turn bay blockage, indicated in Table 16, and these values were adjusted by rounding off to the nearest integer. These results were presented in Table 23:

TABLE 23 Average number of LT cars remaining in the adjacent through lane after blockage

Distribution	The length of left-turn bay					
	3	4	5	6	7	8
Binomial	2.6	2.5	2.5	2.6	2.9	3.3
Poisson	2.7	2.6	2.6	2.8	3.1	3.3
Adjusted value for application	3.0	3.0	3.0	3.0	3.0	3.0

Based on above results and explanation, the values of $N_{S(i,j,k)}$ regarding above two parts were calculated from the Matlab. These values were presented in Table 24 and 25, respectively, and this code was shown in the APPENDIX D.

TABLE 24 The $N_{S(i,j,k)}$ occurred after no left-turn bay blockage situation took place

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
16	5	15503	0	0	0	0	0
	6	69767	54263	0	0	0	0
	7	240311	224807	170543	0	0	0
	8	730625	715121	660857	490313	0	0
	9	2038129	2022625	1968361	1797817	1307503	0
17	5	20348	0	0	0	0	0
	6	94961	74612	0	0	0	0
	7	340118	319769	245156	0	0	0
	8	1075589	1055240	980627	735470	0	0
	9	2383093	2362744	2288131	2042974	2042974	0
18	5	26333	0	0	0	0	0
	6	127280	100946	0	0	0	0
	7	473384	447050	346103	0	0	0
	8	1554959	1528625	1427678	1081574	0	0
	9	4679509	4653175	4552228	4206124	3124549	0
19	5	33648	0	0	0	0	0
	6	168244	134595	0	0	0	0
	7	648944	615295	480699	0	0	0
	8	2211219	2177570	2042974	1562274	0	0
	9	6898044	6864395	6729799	6249099	4686824	0

TABLE 24 Continued.

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
20	**5**	42503	0	0	0	0	0
	6	219603	177099	0	0	0	0
	7	877403	834899	657799	0	0	0
	8	3097478	3054974	2877874	2220074	0	0
	9	10004378	9961874	9784774	9126974	6906899	0
21	**5**	53129	0	0	0	0	0
	6	283359	230229	0	0	0	0
	7	1171389	1118259	888029	0	0	0
	8	4279494	4226364	3996134	3108104	0	0
	9	14294499	14241369	14011139	13123109	10015004	0
22	**5**	65779	0	0	0	0	0
	6	361789	296009	0	0	0	0
	7	1545829	1480049	1184039	0	0	0
	8	5837974	5772194	5476184	4292144	0	0
	9	20145124	20079344	19783334	18599294	14307149	0
23	**5**	80729	0	0	0	0	0
	6	457469	376739	0	0	0	0
	7	2018249	1937519	1560779	0	0	0
	8	7871174	7790444	7413704	5852924	0	0
	9	28031249	27950519	27573779	26012999	20160074	0
24	**5**	98279	0	0	0	0	0
	6	573299	475019	0	0	0	0
	7	2609099	2510819	2035799	0	0	0
	8	10497824	10399544	9924524	7888724	0	0
	9	38546624	38448344	37973324	35937524	28048799	0
25	**5**	118754	0	0	0	0	0
	6	712529	593774	0	0	0	0
	7	3342104	3223349	2629574	0	0	0
	8	13860404	13741649	13147874	10518299	0	0
	9	52427504	52308749	51714974	49085399	38567099	0
26	**5**	142505	0	0	0	0	0
	6	878786	736280	0	0	736280	0
	7	4244642	4102136	3365855	0	4102136	0
	8	18128798	17986292	17250011	13884155	17986292	0
	9	70580054	70437548	69701267	66335411	52451255	0
27	**5**	169910	0	0	0	0	0
	6	1076102	906191	0	0	0	0
	7	5348150	5178239	4272047	0	0	0

TABLE 24 Continued.

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
27	**8**	23504354	23334443	22428251	18156203	0	0
	9	94111814	93941903	93035711	88763663	70607459	0
28	**5**	201375	0	0	0	0	0
	6	1308943	1107567	0	0	0	0
	7	6688559	6487183	5379615	0	0	0
	8	30224379	30023003	28915435	23535819	0	0
	9	124367659	124166283	123058715	117679099	94143279	0
29	**5**	237335	0	0	0	0	0
	6	1582239	1344903	0	0	0	0
	7	8306759	8069423	6724519	0	0	0
	8	38567099	38329763	36984859	30260339	0	0
	9	162970719	162733383	161388479	154663959	124403619	0
30	**5**	278255	0	0	0	0	0
	6	1901415	1623159	0	0	0	0
	7	10249095	9970839	8347679	0	0	0
	8	48857115	48578859	46955699	38608019	0	0
	9	211868755	211590499	209967339	201619659	163011639	0
31	**5**	324631	0	0	0	0	0
	6	2272423	1947791	0	0	0	0
	7	12567895	12243263	10295471	0	0	0
	8	61471387	61146755	59198963	48903491	0	0
	9	273386519	273061887	271114095	260818623	211915131	0

TABLE 25 The $N_{S(i,j,k)}$ occurred after the left-turn bay blockage situation took place

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
16	**2**	135	0	0	0	0	0
	3	951	815	0	0	0	0
	4	4827	4691	3875	0	0	0
	5	20331	20195	19379	15503	0	0
	6	74595	74459	73643	69767	54263	0
	7	245139	245003	244187	240311	224807	170543
	8	735453	735317	734501	730625	715121	660857
	9	2042957	2042821	2042005	2038129	2022625	1968361
17	**2**	152	0	0	0	0	0
	3	1121	968	0	0	0	0
	4	5966	5813	4844	0	0	0

TABLE 25 Continued.

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
17	5	26315	26162	25193	20348	0	0
	6	100928	100775	99806	94961	74612	0
	7	346085	345932	344963	340118	319769	245156
	8	1081556	1081403	1080434	1075589	1055240	980627
	9	3124531	3124378	3123409	3118564	3098215	3023602
18	2	170	0	0	0	0	0
	3	1310	1139	0	0	0	0
	4	7295	7124	5984	0	0	0
	5	33629	33458	32318	26333	0	0
	6	134576	134405	133265	127280	100946	0
	7	480680	480509	479369	473384	447050	346103
	8	1562255	1562084	1560944	1554959	1528625	1427678
	9	4686805	4686634	4685494	4679509	4653175	4552228
19	2	189	0	0	0	0	0
	3	1519	1329	0	0	0	0
	4	8834	8644	7314	0	0	0
	5	42483	42293	40963	33648	0	0
	6	177079	176889	175559	168244	134595	0
	7	657779	657589	656259	648944	615295	480699
	8	2220054	2219864	2218534	2211219	2177570	2042974
	9	6906879	6906689	6905359	6898044	6864395	6729799
20	2	209	0	0	0	0	0
	3	1749	1539	0	0	0	0
	4	10604	10394	8854	0	0	0
	5	53108	52898	51358	42503	0	0
	6	230208	229998	228458	219603	177099	0
	7	888008	887798	886258	877403	834899	657799
	8	3108083	3107873	3106333	3097478	3054974	2877874
	9	10014983	10014773	10013233	10004378	9961874	9784774
21	2	230	0	0	0	0	0
	3	2001	1770	0	0	0	0
	4	12627	12396	10625	0	0	0
	5	65757	65526	63755	53129	0	0
	6	295987	295756	293985	283359	230229	0
	7	1184017	1183786	1182015	1171389	1118259	888029
	8	4292122	4291891	4290120	4279494	4226364	3996134
	9	14307127	14306896	14305125	14294499	14241369	14011139

TABLE 25 Continued.

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
22	**2**	252	0	0	0	0	0
	3	2276	2023	0	0	0	0
	4	14926	14673	12649	0	0	0
	5	80706	80453	78429	65779	0	0
	6	376716	376463	374439	361789	296009	0
	7	1560756	1560503	1558479	1545829	1480049	1184039
	8	5852901	5852648	5850624	5837974	5772194	5476184
	9	20160051	20159798	20157774	20145124	20079344	19783334
23	**2**	275	0	0	0	0	0
	3	2575	2299	0	0	0	0
	4	17525	17249	14949	0	0	0
	5	98255	97979	95679	80729	0	0
	6	474995	474719	472419	457469	376739	0
	7	2035775	2035499	2033199	2018249	1937519	1560779
	8	7888700	7888424	7886124	7871174	7790444	7413704
	9	28048775	28048499	28046199	28031249	27950519	27573779
24		299	0	0	0	0	0
	3	2899	2599	0	0	0	0
	4	20449	20149	17549	0	0	0
	5	118729	118429	115829	98279	0	0
	6	593749	593449	590849	573299	475019	0
	7	2629549	2629249	2626649	2609099	2510819	2035799
	8	10518274	10517974	10515374	10497824	10399544	9924524
	9	38567074	38566774	38564174	38546624	38448344	37973324
25	**2**	324	0	0	0	0	0
	3	3249	2924	0	0	0	0
	4	23724	23399	20474	0	0	0
	5	142479	142154	139229	118754	0	0
	6	736254	735929	733004	712529	593774	0
	7	3365829	3365504	3362579	3342104	3223349	2629574
	8	13884129	13883804	13880879	13860404	13741649	13147874
	9	52451229	52450904	52447979	52427504	52308749	51714974
26	**2**	350	0	0	0	0	0
	3	3275	3275	0	0	0	0
	4	27026	27026	23750	0	0	0
	5	169532	169532	166256	142505	0	0
	6	905813	905813	902537	878786	736280	0
	7	4271669	4271669	4268393	4244642	4102136	3365855
	8	18155825	18155825	18152549	18128798	17986292	17250011
	9	70607081	70607081	70603805	70580054	70437548	69701267

TABLE 25 Continued.

TH (*i*)	LT (*j*)	$N_{S(i,j,3)}$	$N_{S(i,j,4)}$	$N_{S(i,j,5)}$	$N_{S(i,j,6)}$	$N_{S(i,j,7)}$	$N_{S(i,j,8)}$
27	2	377	0	0	0	0	0
	3	4031	3653	0	0	0	0
	4	31436	31058	27404	0	0	0
	5	201347	200969	197315	169910	0	0
	6	1107539	1107161	1103507	1076102	906191	0
	7	5379587	5379209	5375555	5348150	5178239	4272047
	8	23535791	23535413	23531759	23504354	23334443	22428251
	9	94143251	94142873	94139219	94111814	93941903	93035711
28	2	405	0	0	0	0	0
	3	4465	4059	0	0	0	0
	4	35930	35524	31464	0	0	0
	5	237306	236900	232840	201375	0	0
	6	1344874	1344468	1340408	1308943	1107567	0
	7	6724490	6724084	6720024	6688559	6487183	5379615
	8	30260310	30259904	30255844	30224379	30023003	28915435
	9	124403590	124403184	124399124	124367659	124166283	123058715
29	2	434	0	0	0	0	0
	3	4929	4494	0	0	0	0
	4	40889	40454	35959	0	0	0
	5	278225	277790	273295	237335	0	0
	6	1623129	1622694	1618199	1582239	1344903	0
	7	8347649	8347214	8342719	8306759	8069423	6724519
	8	38607989	38607554	38603059	38567099	38329763	36984859
	9	163011609	163011174	163006679	162970719	162733383	161388479
30	2	464	0	0	0	0	0
	3	5424	4959	0	0	0	0
	4	46344	45879	40919	0	0	0
	5	324600	324135	319175	278255	0	0
	6	1947760	1947295	1942335	1901415	1623159	0
	7	10295440	10294975	10290015	10249095	9970839	8347679
	8	48903460	48902995	48898035	48857115	48578859	46955699
	9	211915100	211914635	211909675	211868755	211590499	209967339
31	2	495	0	0	0	0	0
	3	5455	5455	0	0	0	0
	4	51831	51831	46375	0	0	0
	5	376463	376463	371007	324631	0	0
	6	2324255	2324255	2318799	2272423	1947791	0
	7	12619727	12619727	12614271	12567895	12243263	10295471
	8	61523219	61523219	61517763	61471387	61146755	59198963
	9	273438351	273438351	273432895	273386519	273061887	271114095

As it can be seen, Table 24 is about the first scenario that the left-turn bay spillback situation occurs after no left-turn bay blockage situation previously occurred, and Table 25 is regarding the second scenario that the left-turn bay spillback situation takes place with a previous left-turn bay blockage situation. Both results showed that the value of $N_{S(i,j,k)}$ decreases as the length of left-turn bay increases. Also, as the length of left-turn bay increases, $N_{S(i,j,k)}$ should decrease because more left-turn vehicles are required to cause spillback.

As presented in Table 24, $N_{S(i,j,k)}$ is equal to zero when the number of left-turn vehicles, j, arriving at the intersection is smaller than k+2 vehicles. In other words, this case is not able to result in any left-turn bay spillback situations because at least k+2 left-turn vehicles is required to cause a left-turn bay spillback situation. However, Table 25 indicates that $N_{S(i,j,k)}$ is not equal to zero even though the number of left-turn vehicles, j, arriving at the intersection is smaller than k+2. The reason is because of the left-turn blockage that took place previously. That is, since the number of left-turn vehicles, shown in Table 23, stayed on the adjacent through lane at the end of the protected LT green time due to this blockage situation, the left-turn bay spillback situation can occur with fewer number of left-turn cars. So, it is evident that the frequency of left-turn bay spillback situation will increase when there is a previous blockage situation.

By using all the values shown in Tables 22 ~ 25, the probability of the left-turn bay spillback situation, P_{spill}, by the length of left-turn bay was obtained using Eq. (9) and Eq. (11). And these results are described in Table 26 and Figure 9:

TABLE 26 Summary of the P_{spill} according to the length of left-turn bay

		P_{block} (3)	P_{block} (4)	P_{block} (5)	P_{block} (6)	P_{block} (7)	P_{block} (8)
Binomial	First part ($P_{(spill,\ no\ block)}$)	0.4269	0.2594	0.1367	0.0597	0.0180	0.0000
	Second part ($P_{(spill,block)}$)	0.9034	0.7852	0.6126	0.4234	0.2573	0.1357
	Final probability	**0.8300**	**0.7300**	**0.5372**	**0.3378**	**0.1750**	**0.0373**
Poisson	First part ($P_{(spill,\ no\ block)}$)	0.3850	0.2338	0.1231	0.0536	0.0162	0.0000
	Second part ($P_{(spill,block)}$)	0.8170	0.7151	0.5576	0.3852	0.2340	0.1233
	Final probability	**0.7241**	**0.6230**	**0.4368**	**0.2563**	**0.1259**	**0.0313**

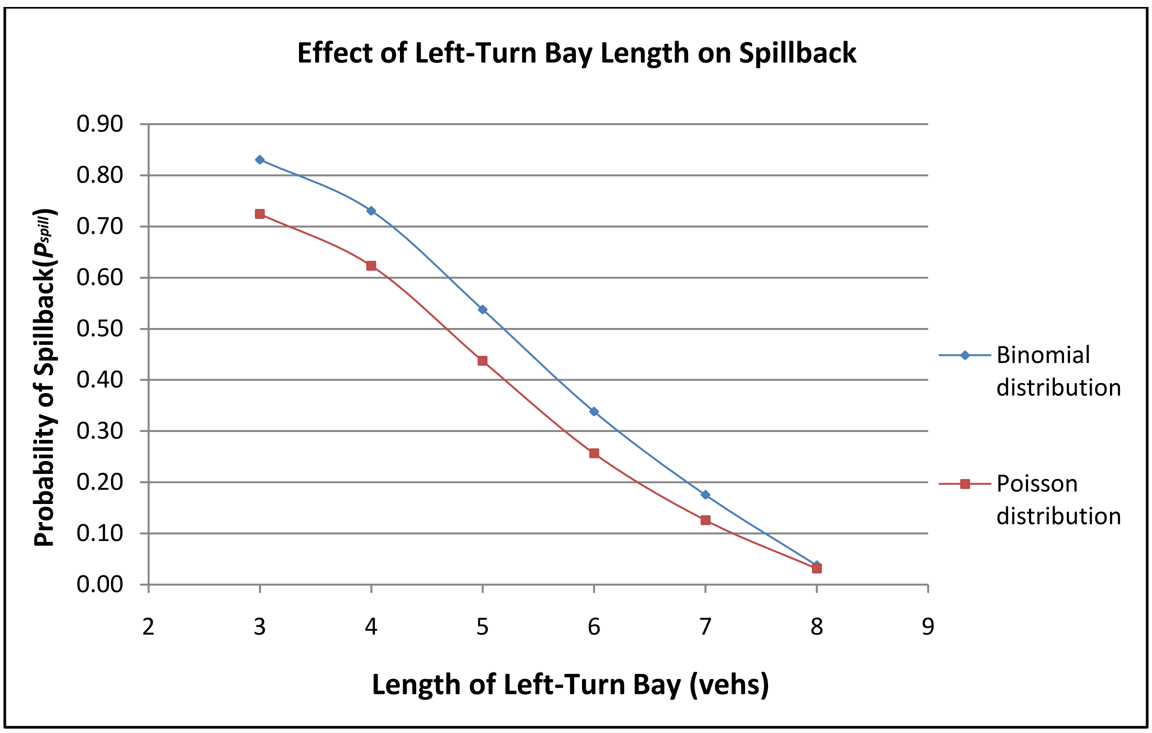

FIGURE 9 Effect of left-turn bay length on the probability of spillback

As shown in Table 26 and Figure 9, it is obvious that the P_{spill} decrease as the length of left-turn bay increases. This means that through movement is more negatively affected by left-turn cars overflowed into the adjacent through lane when the length of the left-turn lane is short.

Also, Figure 9 demonstrates that the P_{spill} in both distributions tend to be reduced in a relatively sharp way when the length of left-turn bay fluctuates between 4 and 7. Here, it is worthwhile to look at when the maximum and minimum reduction occurs according to the change of left-turn bay length. In Table 26, the maximum and minimum reduction of the P_{spill} in the binomial distribution were shown when the length of the left-turn bay extended from 5 to 6 and from 3 to 4, and the maximum and minimum reduction of the P_{spill} in the Poisson distribution were indicated when the length of the left-turn bay extended from 4 to 5 and from 7 to 8. In the binomial and the Poisson distribution, the values of the maximum reduction were 0.1994 and 0.1862, and the values of the minimum reduction are 0.1000 and 0.0946, respectively. These facts provide how much the extension of left-turn bay related to the maximum and minimum reductions have an influence on the P_{spill}.

Furthermore, on average, the P_{spill} of the binomial distribution is higher than the P_{spill} of the Poisson distribution, similar to the case of P_{block}, and the average gap between two distributions is approximately 7.50%. However, when the length of left-turn bay increases, the P_{spill} from the two distributions gradually converge.

CHAPTER V

CAPACITY OF LEFT-TURN AND THROUGH MOVEMENT UNDER LEFT-TURN BLOCKAGE AND SPILLBACK CONDITIONS

This chapter provides reliable estimation of capacity of left-turn and through movement at a signalized intersection under protected leading left-turn signal considering left-turn blockage and spillback. Left-turn and through capacities are estimated using several methods such as the proposed model (using both the binomial distribution and Poisson distribution) and HCM method. In order to prove how well this proposed model reflects the actual traffic situation under leading protected left-turn signalized intersection with a short left-turn bay, model validation process using CORSIM program is performed. Also, this study investigates if a relationship between blockage and spillback indeed exists, and in which conditions (i.e. the length of left-turn bay) this relationship is more significant.

Estimation of the capacity for left-turning and through movement

1) The capacity for left-turning movement

- *Protected left-turn capacity(*$c_{protected}$*)*

In order to more realistically estimate the capacity for left-turning movement, it is required to calculate the protected left-turn capacity by dividing it into two scenarios: the blockage situation and no blockage situation. In order to take into account the left-turn bay blockage situation by through vehicles, it is very important to resolve two issues such as the probability of blockage, P_{block}, due to through traffic and the average number of left-turn vehicles, $E(X_{LT})$, in the left-turn bay when a blockage occurs. So, the results of the P_{block} and $E(X_{LT})$ were already calculated and were presented in Table 20 and Table 21, respectively.

Hence, the protected left-turn capacity can be estimated by the following equation (Zhang and Tong, 2007):

$$c_{protected} = nP_{block}E(X_{LT}) + (1-P_{block})s_{LT}g_{LT}/C \tag{14}$$

where,

n = the number of cycles in a peak hour at the designated intersection

g_{LT} = the effective green interval for the protected left-turn movement

C = the cycle length

s_{LT} = the saturation flow rate for the protected left-turn movement

(This is estimated by HCM method.)

- *Permitted left-turn capacity($c_{permitted}$)*

The permitted left-turn capacity was computed using a method modified from the procedure in APPENDIX C of Chapter 16 in the HCM, and the equations for the permitted left-turn capacity are as follows (Zhang and Tong, 2007):

$$c_{potential} = \frac{V_o e^{-V_g T_C/3600}}{1-e^{-V_g H_f/3600}} \tag{15}$$

$$c_{permitted} = (g_u / C) \times c_{potential} \tag{16}$$

$$g_u = g - g_q \tag{17}$$

$$g_q = \frac{V_R \times R}{s - V_g} \tag{18}$$

where,

$c_{potential}$ = The potential capacity

(the filter saturation flow of permitted left turns as termed in the HCM)

g_u = The unblocked green time (actual green time for the permitted LT)

C = The cycle length

g = The duration of permitted green interval

g_q = The blocked green time (by opposing through traffic)

V_g = The arrival flow rate during the green interval

V_R = The arrival flow rate during the red interval

R = The red interval for through movement

s = The saturation flow rate

T_c = the critical gap for the vehicles turning left, 4.5 seconds

H_f = the follow-up headway for the left-turn vehicles, 2.5 seconds

Therefore, the estimated left-turn capacity according to the length of left-turn bay was summarized in Table 27:

TABLE 27 Summary of the left-turn capacity according to the length of left-turn bay

		The Length of Left-turn bay (vehs)							
		3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Binomial distribution	$c_{protected}$	80	97	118	143	176	213	248	271
	$c_{permitted}$	135	135	135	135	135	135	135	135
	Jumpers	30	30	30	30	30	30	30	30
	Sneakers	30	30	30	30	30	30	30	30
	LT Capacity	**275**	**292**	**313**	**338**	**371**	**408**	**443**	**466**
Poisson distribution	$C_{protected}$	93	110	132	159	182	218	244	264
	$C_{permitted}$	135	135	135	135	135	135	135	135
	Jumpers	30	30	30	30	30	30	30	30
	Sneakers	30	30	30	30	30	30	30	30
	LT Capacity	**288**	**305**	**327**	**354**	**377**	**413**	**439**	**459**
HCM Method	$C_{protected}$	285	285	285	285	285	285	285	285
	$C_{permitted}$	180	180	180	180	180	180	180	180
	LT Capacity	**465**	**465**	**465**	**465**	**465**	**465**	**465**	**465**

As shown in Table 27, one sneaker is assumed to appear every cycle, and one jumper is also assumed to arrive every cycle.

With regard to the permitted left-turn capacity, indicated in Table 27, this capacity is fixed regardless of the distribution and the length of left-turn bay because this capacity is only affected by the opposing through traffic, shown in Eq. (15). Also, in order to get the permitted left-turn capacity, arrival type is assumed to be 3, which is related to the determination of the value of V_g and V_R. The permitted left-turn capacity was calculated to be 135 vph.

Hence, it can be concluded that the difference of total left-turn capacity according to the length of left-turn bay is entirely due to the discrepancy of protected left-turn capacity affected by the value of P_{block} and $E(X_{LT})$.

2) The capacity for through movement:

Similarly to the left-turn capacity, the capacity for through movement is also calculated by considering two scenarios, with left-turn bay spillback and without left-turn bay spillback. Reduction of the capacity for through movement is caused by the left-turn bay spillback situation because the adjacent through lane cannot be used due to the blockage by left-turn vehicles. So, the through capacity is decreased by about one lane, and the through capacity is estimated by the following equation (Zhang and Tong, 2007):

$$
\begin{aligned}
c_{TH} &= P_{spill}\left[N_{TH} * n + (N_{LN} - 1) s_{TH} g_{TH} / C\right] + N_{LN}(1 - P_{spill}) s_{TH} g_{TH} / C \\
&= P_{spill}(N_{TH} * n - s_{TH} g_{TH} / C) + N_{LN} s_{TH} g_{TH} / C
\end{aligned} \tag{19}
$$

where,

n = the number of cycles in a peak hour at the designated intersection

N_{TH} = the number of vehicles arriving on the adjacent through lane before the spillback within each cycle (This value is obtained from CORSIM)

N_{LN} = the number of through lanes in the approach

g_{TH} = the green interval for the adjacent through movement

C = the cycle length

s_{TH} = the actual saturation flow rate for the through movement

(This is estimated by the HCM method.)

So, the estimated left-turn capacity according to the length of left-turn bay was presented in Table 28:

TABLE 28 Summary of the through capacity according to the length of left-turn bay

	The Length of Left-turn bay					
	3	4	5	6	7	8
Binomial distribution	1038	1106	1237	1373	1484	1578
Poisson distribution	1110	1179	1306	1429	1518	1582
HCM Method	1603	1603	1603	1603	1603	1603

Model validation using CORSIM program

This procedure validates how well this new process reflects the actual traffic situation under leading protected left-turn signalized intersection with short left-turn bay. To do so, microscopic traffic simulation program CORSIM was used. The calibration primarily considers the parameters related to queue discharge characteristics and start-up lost time based on several elements such as queue lengths in left-turn lane as well as through lanes (Zhang and Tong, 2007). The researcher compared the capacity results from two methods by varying the length of the left-turn bay, and the results from the CORSIM program obtained by conducting 8 runs for each length scenario to account variability because of CORSIM's stochastic property (Zhang, Y., Zhang, L., and Owen, L, 2001). Additionally, the capacity results from the HCM method were also calculated for the comparison purpose. Figures 10 and 11 indicate the validation for the left-turn capacity model and through capacity model, separately.

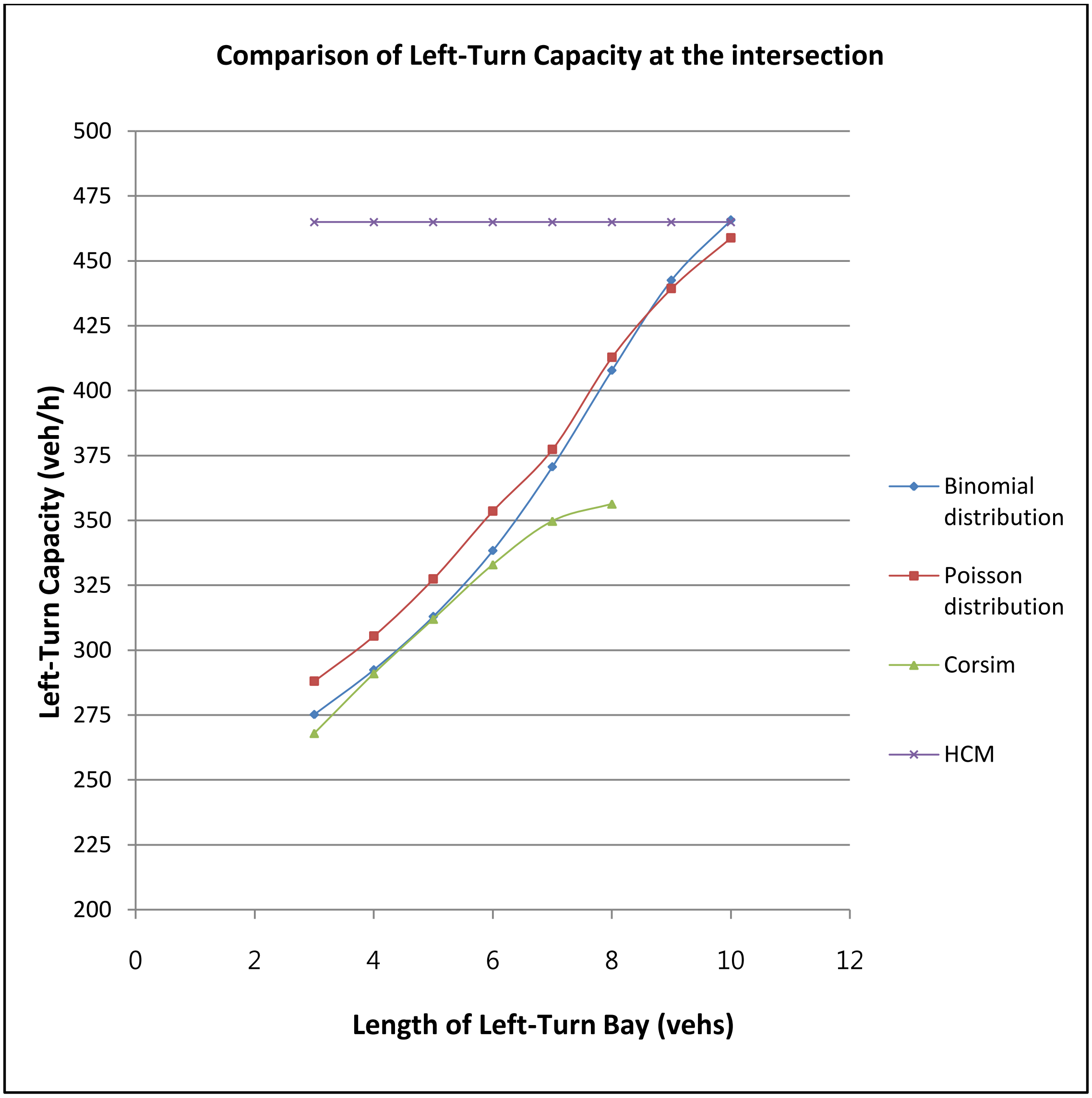

FIGURE 10 Validation of the left-turn capacity model

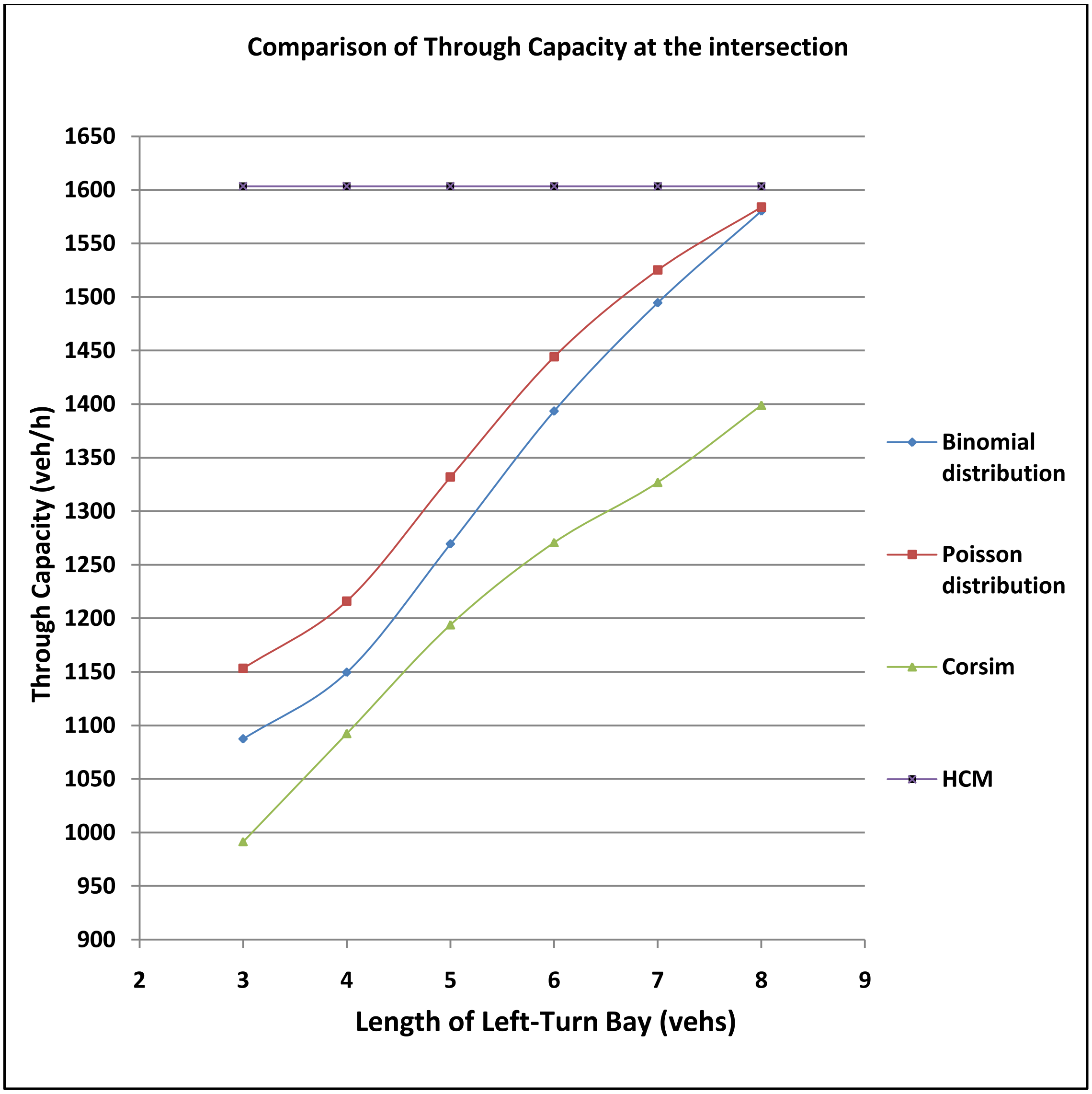

FIGURE 11 Validation of the through capacity model

Figures 10 and 11 demonstrate that both capacity results obtained from the binomial distribution better match the result of CORSIM that reflects on the real-world. On the other hand, both capacity results from the HCM method are constant regardless of the left-turn bay length and tend to overestimate the left-turn and through capacity. The HCM method significantly overestimates the capacity because it deos not consider capacity reduction due to spillback and blockage situations when the length of left-turn bay is short.

Particularly, the estimated left-turn capacity from the binomial distribution better reflects the reduction of left-turn capacity due to the left-turn bay blockage situation when the length of the left-turn bay is from 3 to 6. That is, when the left-turn traffic is often interrupted by the blockage, this estimation method for the left-turn capacity should be better than HCM method and the Poisson distribution. Also, Figure 11 indicates the estimated through capacity from the binomial distribution better matches the through capacity obtained from CORSIM that reflects on the real-world, especially when the length of left-turn bay is 4. However, the comparison of the through capacity results shows that the through capacity produced from the binomial distribution still has a tendency to overrate the through capacity by and large. This can be explained by the fact that the bottleneck arisen from the left-turn bay spillback situation has negative effects, including the reduction of saturation flow rate, on not only the adjacent through lane but also the right through lane which is not considered in this research. The reduction to the saturation flow rate to the unblocked through lane is evident when there are vehicles frequently lane changing from the blocked through lane. Since this method does not take into account this negative effect of capacity reduction of the unblocked through lanes, the capacity gap between the capacity from the binomial distribution and the capacity from the CORSIM seems existed as shown. The reduction of saturation flow rate in unblocked through lane due to blockage due to left-turn spillback in the through lane next to the left-turning lane is an issue that deserves to be investigated, but it's beyond the scope of this thesis. The other reason of this discrepancy is that the blockage situation of the left-turn bay may further increase the probability of left-turn spillback and negatively impact the capacity of through movement. This issue is investigated in the following section.

Analysis for the relationship between blockage and spillback

The left-turn bay spillback situation might take place under the leading protected left-turn signal as well as the lagging one. This is particularly true when the intersection has a short left-turn bay. The left-turn bay spillback situation generally tends to be overlooked in the leading left-turn signal because much attention has been given to the more common problem of left-turn blockage under the leading left signal. The left-turn spillback situation, however, might happen because the ratio of left-turning vehicle tends to be relatively high in the traffic after the occurrence of left-turn bay blockage. This means left-turn vehicles are often discharged at a rate lower than the average capacity (vehicles per cycle) due to the negative effect of blockage, and this in turn contributes to possible spillback in subsequent cycles because of higher than normal left-turn flow. It is very important to investigate if this relationship indeed exists, and in which conditions (i.e. the length of left-turn bay) this relationship is more significant.

Therefore, in order to verify this relationship between left-turn spillback and blockages, the $P_{(spill,\ no\ block)}$ and the $P_{(spill,\ block)}$, previously obtained and shown in Table 26, are compared. Figure 12 demonstrates more clearly how much an effect of the left-turn bay blockage has on left-turn bay spillback at a signalized intersection with a short left-turn bay.

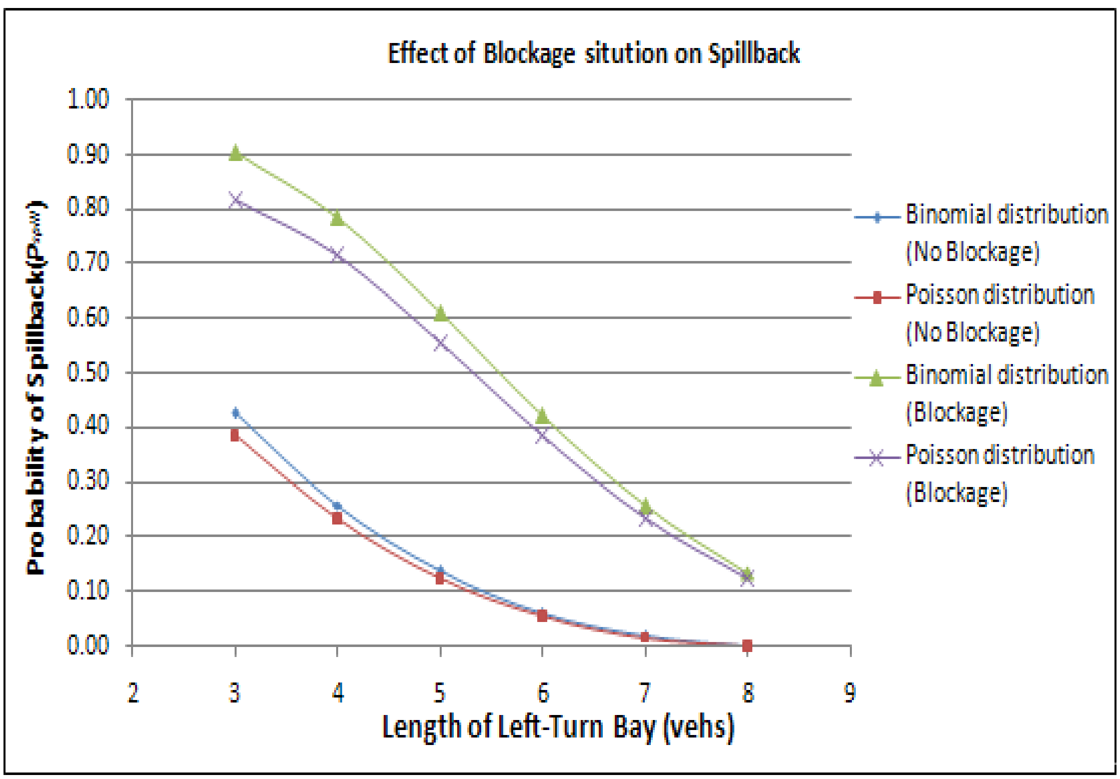

FIGURE 12 The effect of blockage situation on spillback situation (P_{spill})

Figure 12 describes the results of the $P_{(spill,\ block)}$ that are almost twice as the results of the $P_{(spill,\ no\ block)}$. This difference means that the $P_{(spill,\ block)}$ rises considerably compared to the $P_{(spill,\ no\ block)}$ because the left-turn bay spillback situation is significantly affected by the left-turn bay blockage situation, especially when the length of left-turn bay is not long. This result clearly shows that the left-turn bay spillback situation should be considered in order to precisely estimate the capacity for through movement because the left-turn bay blockage situations under leading left-turn signal can increase the probability of left-turn spillback.

Also, Figure 12 indicates that the $P_{(spill,\ block)}$ tends to be more rapidly reduced, while the $P_{(spill,\ no\ block)}$ seems to be steadily decreased as the length of left-turn bay increases. This is because the $P_{(spill,\ block)}$ undergoes a strong influence from the tendency of the P_{block} decreased more steeply as the left-turn lane increases.

Additionally, Figure 13 demonstrates the comparison of the capacities for through movement calculated using the $P_{(spill,\ no\ block)}$ and the $P_{(spill,\ block)}$. The result supports the need to consider the effect of left-turn bay blockage on left-turn bay spillback at a signalized intersection with a short left-turn bay under the leading left-turn signal.

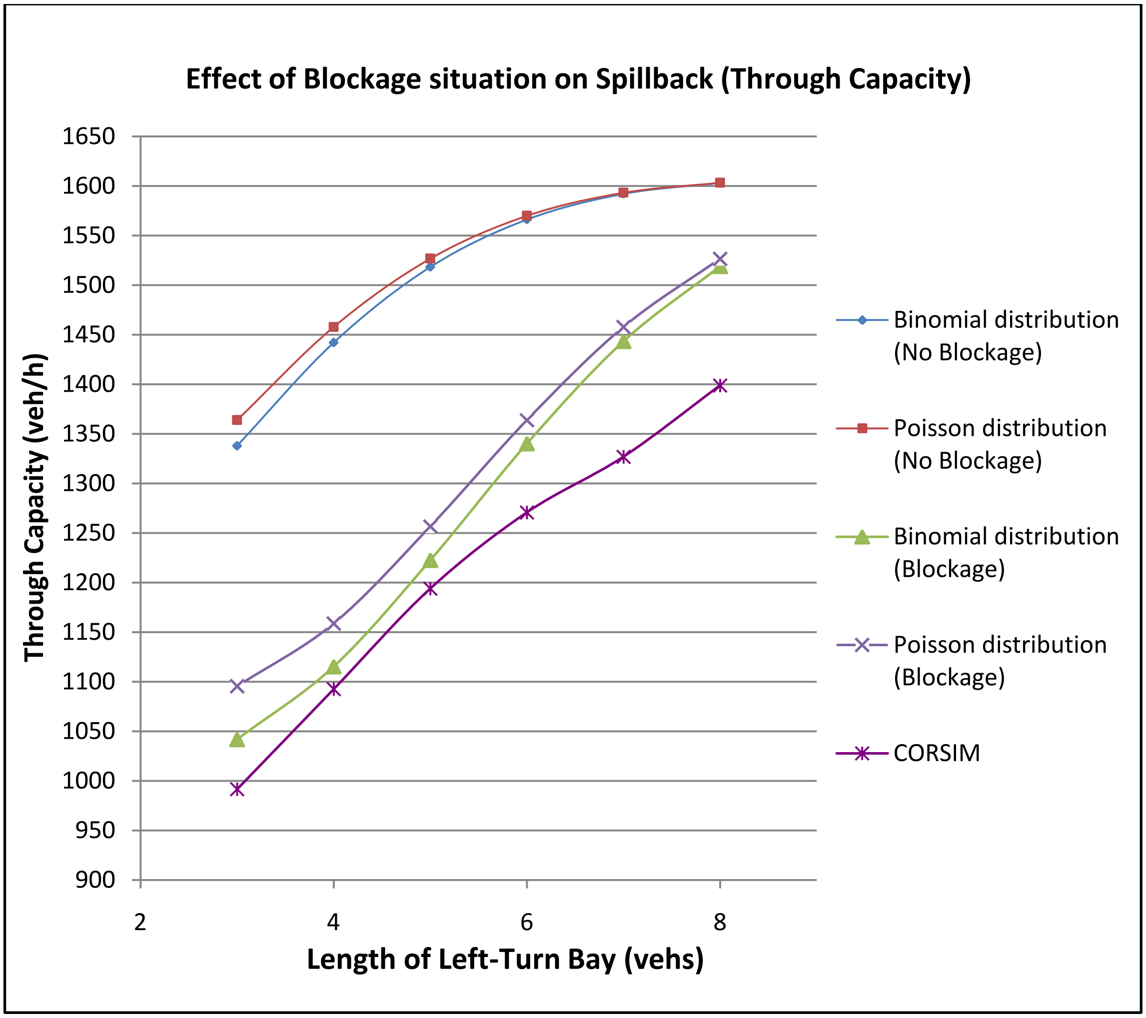

FIGURE 13 The effect of blockage situation on spillback situation (Through capacity)

CHAPTER VI

CONCLUSIONS AND FUTURE WORK

This research developed more realistic models for left-turn and through volume capacity by taking into account the probabilistic nature of the left-turn bay blockages and spillbacks at a signalized intersection under the leading phasing scheme with a short left-turn bay. To do so, the binomial distribution was applied as the arrival distribution for through movement considering the characteristics for expected arrivals under heavy flow conditions. Also, the proposed capacity estimation model for through traffic specifically considered the impact of the left-turn bay spillback situations, and this research ascertained the effect of left-turn bay blockage on left-turn bay spillback situations. The research resulted in the following conclusions:

1. This research demonstrates that the binomial distribution better reflects the arrival distribution of the through vehicles than the Poisson distribution under heavy traffic conditions.

2. The proposed capacity models using the binomial distribution prove to be the most accurate and reliable for estimating the left-turn and through capacity at a signalized intersection in case there are left-turn bay blockages and spillbacks under the leading protected left-turn phasing.

3. This research demonstrates that the length of the left-turn bay has a considerable influence on the capacity for left-turn and through movement because it affects on the number of the left-turn bay blockage cases, $N_{B(i,j,k)}$, or the number of the left-turn bay spillback cases, $N_{S(i,j,k)}$, which determine the probability of left-turn bay blockage, P_{block},

or the probability of left-turn bay spillback, P_{spill}. It is evident that this research makes up for the weak point of HCM methodology that does not take into consideration the effect of the length of the left-turn bay.

4. In order to more precisely estimate the through capacity at a signalized intersection with a short left-turn bay under leading left-turn signal, it is very important to practically calibrate the P_{spill}. Since many left-turn bay blockages occur as the length of the left-turn bay is getting shorter, left-turn bay blockages should be also considered to calculate the P_{spill}. This research shows that it is obvious that the left-turn bay blockages are critical to the left-turn bay spillbacks, and more reasonable values of the P_{spill} is produced by applying this relationship into the equation for the P_{spill}.

Future work

1. Although the proposed capacity estimation models using the binomial distribution better estimate the left-turn and adjacent through capacity than previous studies, more factors such as pedestrians and consideration of mixed vehicle types are necessary to be analyzed and discussed for more practical and accurate models.

2. In the proposed model for through movement, the reduction of the saturation flow rate for the through traffics due to the left-turn bay spillback situations requires to be further investigated. With the results from this future study, through capacity will be more precisely estimated using the methods developed in this thesis by applying more reasonable saturation flow rate for the through movement.

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APPENDIX A

1. Arrival data for the traffic condition of a low range (Unit: vehicles / 10s)

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
1	2	2	0
2	3	2	1
3	1	1	0
4	5	5	0
5	3	2	1
6	1	1	0
7	1	1	0
8	3	2	1
9	4	4	0
10	2	2	0
11	5	5	0
12	3	3	0
13	0	0	0
14	4	3	1
15	3	3	0
16	2	2	0
17	1	1	0
18	0	0	0
19	4	4	0
20	3	2	1
21	4	4	0
22	5	5	0
23	0	0	0
24	4	3	1
25	1	1	0
26	1	1	0
27	2	2	0
28	0	0	0
29	4	3	1
30	5	5	0
31	1	1	0
32	4	3	1
33	1	0	1
34	3	2	1

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
35	1	1	0
36	3	2	1
37	4	4	0
38	3	3	0
39	4	4	0
40	3	3	0
41	3	3	0
42	2	2	0
43	2	2	0
44	1	1	0
45	1	1	0
46	3	3	0
47	4	4	0
48	2	1	1
49	5	4	1
50	0	0	0
51	3	2	1
52	4	4	0
53	0	0	0
54	1	1	0
55	3	3	0
56	3	3	0
57	1	1	0
58	4	4	0
59	3	3	0
60	3	3	0
61	2	2	0
62	3	2	1
63	2	2	0
64	1	1	0
65	5	4	1
66	1	1	0
67	3	2	1
68	2	2	0
69	4	3	1
70	1	1	0
71	1	1	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
72	4	4	0
73	4	4	0
74	2	2	0
75	5	5	0
76	2	1	1
77	1	1	0
78	2	1	1
79	2	1	1
80	4	2	2
81	4	4	0
82	2	2	0
83	0	0	0
84	4	3	1
85	1	1	0
86	1	1	0
87	4	4	0
88	5	4	1
89	1	1	0
90	2	1	1
91	3	3	0
92	2	2	0
93	2	2	0
94	1	1	0
95	3	2	1
96	5	4	1
97	3	3	0
98	1	0	1
99	2	1	1
100	3	3	0
101	4	4	0
102	0	0	0
103	7	7	0
104	0	0	0
105	4	4	0
106	2	2	0
107	2	2	0
108	4	3	1

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
109	3	2	1
110	2	2	0
111	2	2	0
112	2	2	0
113	4	2	2
114	4	4	0
115	1	1	0
116	3	2	1
117	4	3	1
118	4	4	0
119	2	2	0
120	2	2	0
121	2	2	0
122	0	0	0
123	2	1	1
124	6	5	1
125	2	1	1
126	3	3	0
127	0	0	0
128	4	2	2
129	4	4	0
130	1	1	0
131	4	3	1
132	1	1	0
133	3	3	0
134	3	3	0
135	1	1	0
136	5	4	1
137	1	1	0
138	2	2	0
139	0	0	0
140	3	3	0
141	0	0	0
142	2	2	0
143	5	5	0
144	3	3	0
145	2	2	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
146	2	2	0
147	3	3	0
148	2	1	1
149	4	4	0
150	3	3	0
151	3	1	2
152	2	1	1
153	3	3	0
154	1	1	0
155	6	4	2
156	2	2	0
157	3	3	0
158	1	1	0
159	1	1	0
160	4	4	0
161	3	2	1
162	2	2	0
163	3	2	1
164	3	3	0
165	0	0	0
166	4	3	1
167	3	3	0
168	3	3	0
169	4	4	0
170	2	1	1
171	4	3	1
172	4	4	0
173	2	2	0
174	2	2	0
175	2	2	0
176	3	3	0
177	1	1	0
178	3	3	0
179	0	0	0
180	3	1	2
181	5	5	0
182	1	1	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
183	3	3	0
184	5	4	1
185	0	0	0
186	3	3	0
187	4	3	1
188	3	3	0
189	2	2	0
190	6	6	0
191	0	0	0
192	4	4	0
193	1	1	0
194	0	0	0
195	3	3	0
196	3	3	0
197	3	3	0
198	3	1	2
199	1	1	0
200	5	4	1
201	1	1	0
202	3	3	0
203	2	2	0
204	2	1	1
205	3	3	0
206	1	1	0
207	5	5	0
208	2	1	1
209	3	2	1
210	4	3	1
211	1	1	0
212	1	1	0
213	1	1	0
214	4	4	0
215	3	3	0
216	3	3	0
217	2	2	0
218	3	3	0
219	1	1	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
220	4	3	1
221	3	3	0
222	3	3	0
223	2	1	1
224	3	3	0
225	2	2	0
226	0	0	0
227	4	4	0
228	2	2	0
229	1	0	1
230	4	2	2
231	5	4	1
232	3	3	0
233	0	0	0
234	0	0	0
235	3	1	2
236	3	2	1
237	2	2	0
238	1	1	0
239	3	3	0
240	4	4	0
241	3	3	0
242	1	1	0
243	4	3	1
244	5	5	0
245	0	0	0
246	2	2	0
247	3	3	0
248	3	1	2
249	4	3	1
250	1	1	0
251	2	2	0
252	3	2	1
253	2	2	0
254	4	4	0
255	2	2	0
256	2	1	1

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
257	2	2	0
258	5	5	0
259	2	2	0
260	1	1	0
261	7	6	1
262	0	0	0
263	0	0	0
264	3	3	0
265	5	3	2
266	4	4	0
267	3	3	0
268	0	0	0
269	0	0	0
270	3	2	1
271	2	1	1
272	1	1	0
273	5	5	0
274	3	3	0
275	1	1	0
276	3	2	1
277	1	1	0
278	3	2	1
279	5	4	1
280	1	1	0
281	1	1	0
282	5	4	1
283	3	3	0
284	4	4	0
285	2	2	0
286	2	2	0
287	0	0	0
288	2	0	2
289	2	1	1
290	3	3	0
291	3	3	0
292	4	2	2
293	1	1	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
294	3	2	1
295	4	3	1
296	2	2	0
297	1	1	0
298	4	4	0
299	1	0	1
300	4	4	0
301	1	0	1
302	3	3	0
303	3	3	0
304	3	3	0
305	2	2	0
306	2	1	1
307	1	1	0
308	3	3	0
309	0	0	0
310	6	6	0
311	4	4	0
312	3	3	0
313	3	2	1
314	3	2	1
315	3	3	0
316	1	1	0
317	3	3	0
318	4	4	0
319	2	1	1
320	3	3	0
321	4	3	1
322	3	2	1
323	1	1	0
324	2	2	0
325	5	4	1
326	2	2	0
327	3	2	1
328	1	1	0
329	3	3	0
330	4	3	1

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
331	1	1	0
332	2	2	0
333	6	6	0
334	1	1	0
335	2	2	0
336	2	2	0
337	1	1	0
338	5	5	0
339	5	4	1
340	0	0	0
341	4	4	0
342	7	7	0
343	1	1	0
344	3	2	1
345	3	3	0
346	1	1	0
347	0	0	0
348	3	2	1
349	1	1	0
350	3	3	0
351	4	4	0
352	1	1	0
353	2	2	0
354	1	1	0
355	6	4	2
356	2	2	0
357	2	2	0
358	3	2	1
359	6	6	0
360	0	0	0

2. Arrival data for the traffic condition of a high range (Unit: vehicles / 10s)

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
1	6	4	2
2	5	5	0
3	6	5	1

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
4	4	4	0
5	4	3	1
6	5	5	0
7	4	4	0
8	6	3	3
9	3	3	0
10	5	5	0
11	7	7	0
12	5	5	0
13	4	4	0
14	5	4	1
15	5	4	1
16	5	4	1
17	4	3	1
18	7	6	1
19	6	6	0
20	5	5	0
21	5	3	2
22	3	3	0
23	10	7	3
24	6	5	1
25	5	3	2
26	6	5	1
27	4	3	1
28	5	5	0
29	6	6	0
30	4	3	1
31	4	3	1
32	6	4	2
33	4	4	0
34	5	2	3
35	6	6	0
36	6	5	1
37	5	4	1
38	7	7	0
39	4	2	2
40	5	5	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
41	3	2	1
42	7	7	0
43	6	5	1
44	6	6	0
45	3	3	0
46	3	4	1
47	10	8	2
48	5	3	2
49	5	5	0
50	2	2	0
51	7	7	0
52	6	6	0
53	2	1	1
54	6	5	1
55	5	4	1
56	7	5	2
57	4	3	1
58	4	4	0
59	6	5	1
60	6	4	2
61	6	5	1
62	6	6	0
63	4	4	0
64	4	4	0
65	5	5	0
66	5	4	1
67	7	5	2
68	5	5	0
69	4	3	1
70	4	3	1
71	8	7	1
72	7	5	2
73	6	5	1
74	5	4	1
75	5	5	0
76	5	5	0
77	6	4	2

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
78	5	4	1
79	6	6	0
80	5	3	2
81	4	3	1
82	5	4	1
83	8	8	0
84	7	6	1
85	4	3	1
86	7	7	0
87	4	4	0
88	4	4	0
89	5	4	1
90	6	6	0
91	4	3	1
92	5	4	1
93	2	2	0
94	3	1	2
95	9	9	0
96	6	4	2
97	9	7	2
98	5	5	0
99	5	5	0
100	4	4	0
101	7	5	2
102	5	5	0
103	3	2	1
104	0	0	0
105	4	3	1
106	12	8	4
107	10	7	3
108	6	5	1
109	4	3	1
110	3	2	1
111	5	5	0
112	6	6	0
113	3	3	0
114	6	6	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
115	4	4	0
116	4	2	2
117	5	2	3
118	8	7	1
119	6	6	0
120	5	4	1
121	4	4	0
122	5	5	0
123	4	4	0
124	6	5	1
125	7	4	3
126	7	5	2
127	3	3	0
128	6	6	0
129	3	2	1
130	5	4	1
131	6	4	2
132	6	5	1
133	6	5	1
134	5	5	0
135	5	4	1
136	5	2	3
137	6	6	0
138	2	2	0
139	3	3	0
140	9	5	4
141	7	5	2
142	7	5	2
143	6	6	0
144	5	4	1
145	7	6	1
146	5	5	0
147	3	3	0
148	6	6	0
149	3	3	0
150	9	5	4
151	5	3	2

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
152	7	6	1
153	5	4	1
154	8	7	1
155	6	5	1
156	6	6	0
157	4	4	0
158	5	5	0
159	7	6	1
160	4	4	0
161	6	5	1
162	1	1	0
163	7	6	1
164	8	7	1
165	7	5	2
166	6	5	1
167	6	5	1
168	6	5	1
169	6	6	0
170	5	5	0
171	4	4	0
172	3	3	0
173	5	4	1
174	3	2	1
175	5	3	2
176	7	6	1
177	8	6	2
178	4	4	0
179	3	3	0
180	7	4	3
181	6	5	1
182	3	3	0
183	6	5	1
184	8	7	1
185	4	3	1
186	5	3	2
187	4	3	1
188	6	6	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
189	6	6	0
190	6	4	2
191	7	7	0
192	5	3	2
193	7	6	1
194	6	5	1
195	3	3	0
196	4	3	1
197	6	4	2
198	4	2	2
199	7	7	0
200	4	4	0
201	7	6	1
202	5	4	1
203	5	5	0
204	5	4	1
205	3	3	0
206	5	5	0
207	6	5	1
208	6	4	2
209	4	3	1
210	5	5	0
211	4	3	1
212	5	4	1
213	8	8	0
214	4	4	0
215	7	6	1
216	5	4	1
217	5	4	1
218	5	5	0
219	5	4	1
220	4	4	0
221	7	6	1
222	4	2	2
223	6	5	1
224	6	6	0
225	5	4	1

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
226	6	4	2
227	3	2	1
228	4	3	1
229	7	6	1
230	6	5	1
231	4	3	1
232	4	4	0
233	7	4	3
234	5	4	1
235	6	6	0
236	3	3	0
237	7	6	1
238	4	4	0
239	4	4	0
240	6	5	1
241	4	3	1
242	6	6	0
243	4	4	0
244	3	2	1
245	3	3	0
246	7	6	1
247	4	2	2
248	9	7	2
249	7	6	1
250	4	2	2
251	7	5	2
252	4	3	1
253	4	3	1
254	3	3	0
255	4	4	0
256	3	3	0
257	4	4	0
258	8	4	4
259	9	4	5
260	9	8	1
261	6	4	2
262	3	3	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
263	6	5	1
264	5	5	0
265	4	4	0
266	6	6	0
267	4	4	0
268	3	3	0
269	4	4	0
270	8	3	5
271	5	4	1
272	10	9	1
273	6	6	0
274	7	6	1
275	6	6	0
276	6	5	1
277	4	2	2
278	6	4	2
279	3	2	1
280	7	7	0
281	5	5	0
282	4	2	2
283	4	3	1
284	9	5	4
285	7	6	1
286	4	4	0
287	4	4	0
288	8	7	1
289	3	2	1
290	4	4	0
291	6	6	0
292	6	6	0
293	5	4	1
294	6	4	2
295	3	2	1
296	8	7	1
297	7	6	1
298	5	4	1
299	6	6	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
300	7	7	0
301	5	5	0
302	6	6	0
303	4	3	1
304	3	1	2
305	5	4	1
306	6	3	3
307	6	3	3
308	7	6	1
309	6	6	0
310	4	2	2
311	5	5	0
312	4	4	0
313	8	8	0
314	5	5	0
315	6	5	1
316	4	4	0
317	6	4	2
318	2	2	0
319	4	1	3
320	8	8	0
321	8	8	0
322	6	6	0
323	3	2	1
324	6	6	0
325	6	6	0
326	5	4	1
327	6	6	0
328	4	4	0
329	4	3	1
330	4	2	2
331	1	0	1
332	9	8	1
333	9	6	3
334	5	4	1
335	4	3	1
336	6	6	0

	The total # of vehicles	The # of Through vehicles	The # of Left-turn vehicles
337	7	6	1
338	5	5	0
339	6	5	1
340	3	3	0
341	0	0	0
342	1	1	0
343	9	8	1
344	13	10	3
345	10	8	2
346	0	0	0
347	7	6	1
348	5	5	0
349	5	4	1
350	5	5	0
351	3	1	2
352	0	0	0
353	3	3	0
354	11	9	2
355	6	5	1
356	5	4	1
357	7	6	1
358	6	5	1
359	5	4	1
360	11	9	2

APPENDIX B

Arrival data for the real-world (Unit: vehicles / 10s)

	The # of vehicles		The # of vehicles		The # of vehicles		The # of vehicles
1	8	35	2	69	8	103	6
2	7	36	3	70	6	104	6
3	6	37	5	71	4	105	4
4	5	38	3	72	5	106	6
5	3	39	4	73	4	107	8
6	3	40	6	74	3	108	8
7	3	41	8	75	3	109	5
8	2	42	7	76	6	110	6
9	2	43	8	77	5	111	6
10	2	44	3	78	4	112	4
11	5	45	6	79	5	113	3
12	4	46	4	80	7	114	3
13	6	47	5	81	6	115	7
14	5	48	3	82	7	116	5
15	7	49	2	83	8	117	3
16	6	50	5	84	6	118	4
17	4	51	4	85	5	119	4
18	1	52	6	86	3	120	5
19	2	53	5	87	3	121	5
20	0	54	5	88	2	122	4
21	2	55	7	89	7	123	4
22	2	56	9	90	5	124	4
23	3	57	6	91	2	125	4
24	4	58	4	92	4	126	5
25	4	59	8	93	7	127	5
26	5	60	3	94	8	128	5
27	4	61	2	95	7	129	7
28	4	62	4	96	5	130	4
29	6	63	7	97	5	131	5
30	7	64	6	98	6	132	5
31	5	65	3	99	3	133	3
32	4	66	6	100	3	134	8
33	3	67	8	101	3	135	7
34	3	68	8	102	4	136	5

	The # of vehicles		The # of vehicles		The # of vehicles		The # of vehicles
137	2	176	5	215	5		
138	2	177	6	216	6		
139	2	178	3	217	5		
140	4	179	5	218	5		
141	5	180	5	219	6		
142	5	181	3	220	5		
143	5	182	5	221	6		
144	4	183	6	222	7		
145	6	184	4	223	4		
146	6	185	5	224	4		
147	8	186	6	225	0		
148	6	187	5	226	2		
149	4	188	3	227	3		
150	6	189	1	228	5		
151	4	190	4	229	6		
152	3	191	2	230	3		
153	4	192	5	231	4		
154	6	193	6	232	5		
155	4	194	6	233	7		
156	4	195	5	234	6		
157	4	196	7	235	6		
158	5	197	5	236	5		
159	4	198	4	237	4		
160	8	199	3	238	4		
161	7	200	5				
162	4	201	1				
163	3	202	1				
164	6	203	3				
165	4	204	5				
166	4	205	6				
167	4	206	8				
168	5	207	9				
169	6	208	6				
170	6	209	5				
171	5	210	5				
172	4	211	5				
173	5	212	4				
174	5	213	3				
175	6	214	3				

APPENDIX C

Matlab code used to calculate the value of the $N_{B(i,j,k)}$

```
LT_bay = LT_length+LT_Transition

for TH = 13:29,
    for LT = 2:8,
        for LT_bay = 5:12,

foutput = sprintf('Output/output_TH_%d_LT_%d_Lb_%d.txt', TH, LT, LT_bay)
fid = fopen(foutput, 'w')

N = TH + LT
blockage = zeros(1, N)

count = zeros(1,LT+1);

for ind = 1:LT,
    blockage(ind) = 1;
end;

blockage

number=LT;
while number>0

if blockage(N)>0, blockage(N)=0;
else
    for ind = N:-1:1
        blockage(ind)=1;
        blockage(ind-1)=blockage(ind-1)-1;
        if blockage(ind-1)==0, break;
        end;
    end;
end;

number = 0;
for ind = 1:N
    if blockage (ind)>0, number = number+1;
    end;
end;

    if number==LT,
        blockage;
        LT_number = 0;
        TH_number = 0;
        for ind = 1: N
```

```
            if blockage(ind)==1, LT_number=LT_number+1;
            else TH_number=TH_number+1;
            end;
            if TH_number==LT_bay*2,
              if (LT_number < LT),
                 count(LT_number+1) = count(LT_number+1) + 1;
                    %sprintf('found blockage')
                    %blockage
                end
                break;
            end;
        end;
    end;
end

blockage

total=factorial(N)/factorial(LT)/factorial(TH)

for ind =1:LT+1,
  count(ind)
  count(ind)/total
  fprintf(fid, '%d %d %.8f\n', ind-1, count(ind), count(ind)/total)
end

fclose(fid)
        end
    end
end
```

APPENDIX D

Matlab code used to calculate the value of the $N_{S(i,j,k)}$

```
LT_bay = LT_length+LT_Transition

for TH = 16:31,
    for LT = 5:9,
        for LT_bay = 5:10,

foutput = sprintf('Output/output_TH_%d_LT_%d_Lb_%d.txt', TH, LT, LT_bay)
fid = fopen(foutput, 'w')

N = TH + LT
spillback = zeros(1, N)

count = zeros(1,LT+1);

for ind = 1:LT,
    spillback(ind) = 1;
end;

spillback

number=LT;
while number>0

if spillback(N)>0, spillback(N)=0;
else
    for ind = N:-1:1
        spillback(ind)=1;
        spillback(ind-1)=spillback(ind-1)-1;
        if spillback(ind-1)==0, break;
        end;
    end;
end;

number = 0;
for ind = 1:N
    if spillback (ind)>0, number = number+1;
    end;
end;
    if number==LT,
        spillback;
        LT_number = 0;
        TH_number = 0;
        for ind = 1: N
            if spillback(ind)==1, LT_number=LT_number+1;
```

```
            else TH_number=TH_number+1;
            end;
            if LT_number>LT_bay-1,
              if (TH_number < TH),
                 count(LT_number+1) = count(LT_number+1) + 1;
                    %sprintf('found spillback')
                    %spillback
                end
                break;
            end;
        end;
    end;
end

spillback
total=factorial(N)/factorial(LT)/factorial(TH)

for ind =1:LT+1,
  count(ind)
  count(ind)/total
  fprintf(fid, '%d %d %.8f\n', ind-1, count(ind), count(ind)/total)
end

fclose(fid)
        end
    end
end
```

VITA

Name: Kyoungmin Cho

Address: Department of Civil Engineering

c/o Dr. Y. Zhang

Texas A&M University

College Station, Texas 77843-3136

Email Address: c14624@neo.tamu.edu

Education: B.S., Mechanical Engineering, Korea Military Academy, Seoul Korea

M.S. Civil Engineering, Texas A&M University, 2009